CSE 311: Foundations of Computing

Fall 2014

Lecture 26: Pattern matching, Halting problem

highlights

- DFAs ≡ Regular Expressions
  - No need to know details of NFAs→RegExpressions
- Method for proving no DFAs for languages
  - e.g. $\{0^n1^n : n \geq 0\}$, {Binary palindromes}

pattern matching

- Given
  - a string, **s**, of **n** characters
  - a pattern, **p**, of **m** characters
  - usually **m<<n**
- Find
  - all occurrences of the pattern **p** in the string **s**
- Obvious algorithm:
  - try to see if **p** matches at each of the positions in **s**

    stop at a failed match and try the next position

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

pattern **p** = x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

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string **s** = x y x x y x y x y y x y x y x y y x y x y x x

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x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

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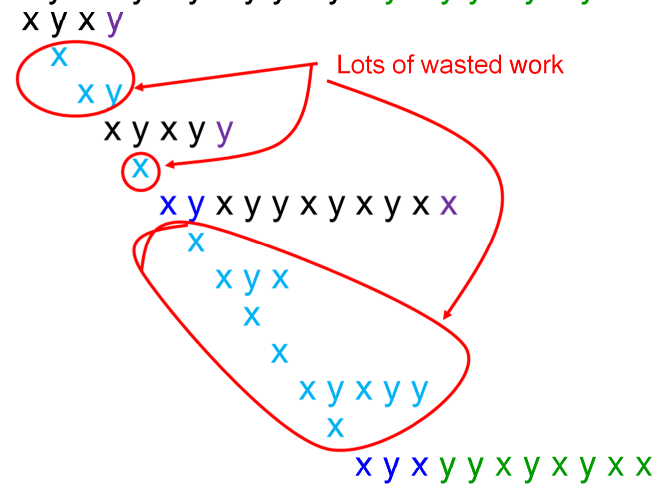
Worst-case time O(**mn**)

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

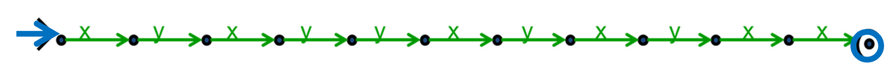


better pattern matching via finite automata

- Build a DFA for the pattern (preprocessing) of size O(**m**)
  - Keep track of the 'longest match currently active'
  - The DFA will have only **m**+1 states
- Run the DFA on the string **n** steps
- Obvious construction method for DFA will be O($\mathbf{m}^2$) but can be done in O(**m**) time.
- Total O(**m**+**n**) time

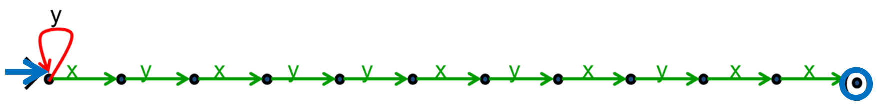
building a DFA for the pattern

pattern **p**=x y x y y x y x y x x



preprocessing the pattern

pattern **p**=x y x y y x y x y x x



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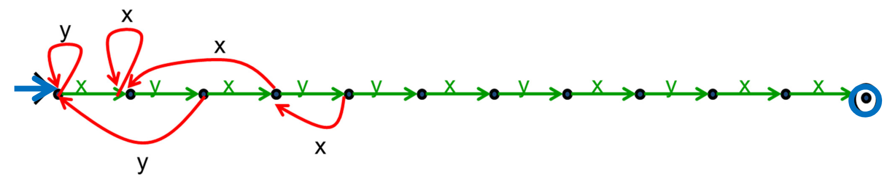

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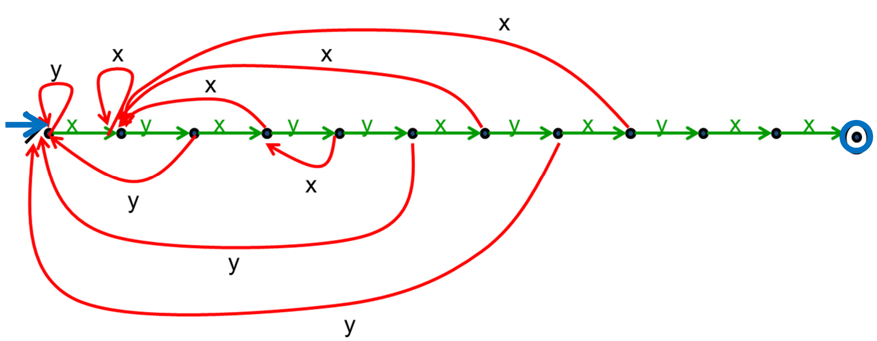

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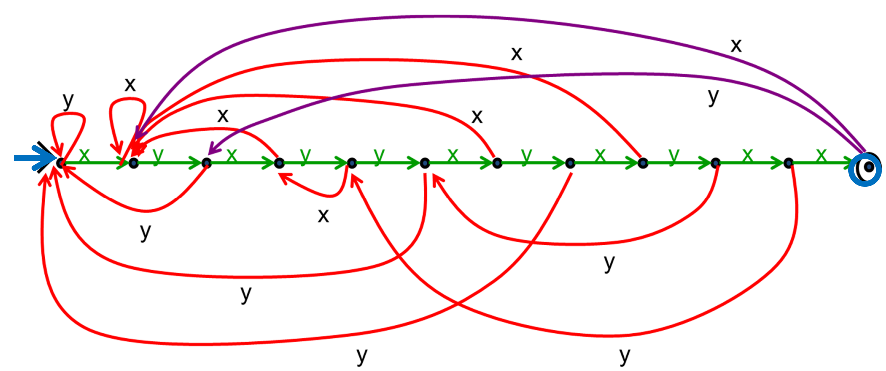
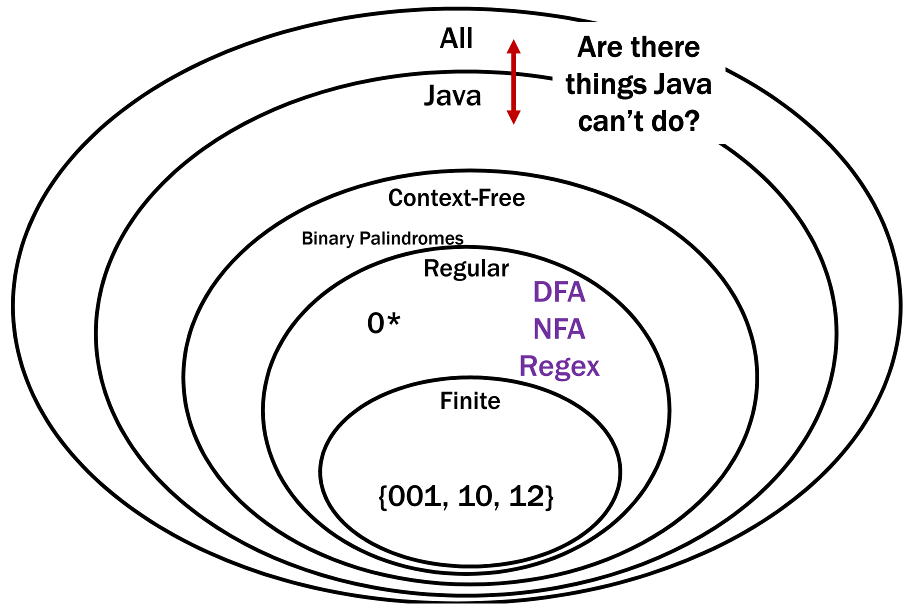


generalizing

- Can search for arbitrary combinations of patterns
  - Not just a single pattern
  - Build NFA for pattern then convert to DFA 'on the fly'.

    Compare DFA constructed above with subset construction for the obvious NFA.

Languages and Machines!



An Assignment Too Simple for 142.

Students should write a Java program that…

- Prints "Hello" to the console
- Eventually exits

GradeIt, PracticeIt, etc. need to grade the students.

How do we write that grading program?

Follow Up Question

What does this program do?

```
_(__,___,____){___/__<=1?_(__,___+1,___
_):!(___%__)?_(__,___+1,0):___%__==___ /
__&&!____?(printf("%d\t",___/__),_(__,_
__+1,0)):___%__>1&&___%__<___/__?_(
__,1+
___,____+!(___/__%(___%__))):___<__*__
?_(__,___+1,____):0;}main(){_(100,0,0);}
```

Sneak Peak

It turns out the simple autograder is impossible to write…

And we'll prove it!

Some Notation and Starting Ideas

We're going to be talking about *Java code* a lot.

CODE(**P**) will mean "the code of the program P"

So, consider the following function:

```
public String P(String x) {
    return new String(Arrays.sort(x.toCharArray());
}
```

What is **P**(CODE(**P**))?

"((()))..;AACPSSaaabceeggghiiiilnnnnnooprrrrrrrrrrrsssttttttuuwxxyy{}"

The Halting Problem

Given: - CODE(**P**) for any program **P**

- input **x**

Output: **true** if **P** halts on input **x**

false if **P** does not halt on input **x**

The Halting Problem

Given: - CODE(**P**) for any program **P**

- input **x**

Output: **true** if **P** halts on input **x**

false if **P** does not halt on input **x**

It turns out that it isn't possible to write a program that solves the Halting Problem.

Proof by contradiction

- **Suppose that H is a Java program that solves the Halting problem. Then we can write this program:**

```
public static void D(x) {
   if (H(x,x) == true) {
      while (true);   /* don't halt */
   }
   else {
      return;         /*    halt    */
   }
}
```

- **Does D**(CODE(**D**)) halt?

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public static void D(x) {
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H solves the halting problem implies that
H(CODE(**D**),x) is **true** iff **D**(x) halts, H(CODE(**D**),x) is **false** iff not

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H solves the halting problem implies that
H(CODE(**D**),x) is **true** iff **D**(x) halts, H(CODE(**D**),x) is **false** iff not

Suppose **D**(CODE(**D**)) **halts**.
Then, we must be in the **second** case of the if.
So, H(CODE(**D**), CODE(**D**)) is **false**
Which means **D**(CODE(**D**)) **doesn't halt**

Does **D**(CODE(**D**)) halt?

```
public static void D(x) {
   if (H(x,x) == true) {
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Which means **D**(CODE(**D**)) **doesn't halt**

Suppose **D**(CODE(**D**)) **doesn't halt**.
Then, we must be in the **first** case of the if.
So, H(CODE(**D**), CODE(**D**)) is **true**.
Which means **D**(CODE(**D**)) **halts**.

Does **D**(CODE(**D**)) halt?

```
public static void D(x) {
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Suppose **D**(CODE(**D**)) **halts**.
Then, we must be in the **second** case of the if.
So, H(CODE(**D**), CODE(**D**)) is **false**
Which means **D**(CODE(**D**)) **doesn't halt**

Suppose **D**(CODE(**D**)) **doesn't halt**.
Then, we must be in the **first** case of the if.
So, H(CODE(**D**), CODE(**D**)) is **true**.
Which means **D**(CODE(**D**)) **halts**.

Contradiction!

That's it!

- **We proved that there is no computer program that can solve the Halting Problem.**
  - **There was nothing special about Java**
- **This tells us that there is no compiler that can check our programs and guarantee to find any infinite loops they might have.**

What's next?

- **We showed: If some "hypothetical" subroutine H existed that solved the Halting Problem then it would let us build a program D that cannot possibly exist**
  - **We will use the same idea to show that programs solving other problems are impossible, but we now will be able to use that H cannot exist**
- **A key piece of the proof was considering what a program does when given its own code as input**
  - **This was inspired by a method to compare the sizes of infinite sets call *diagonalization* that we will study next class**