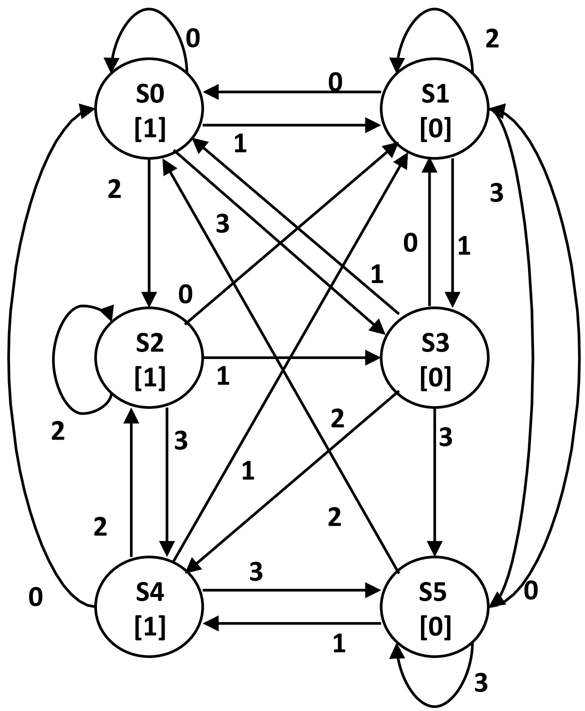
CSE 311: Foundations of Computing

Fall 2014

Lecture 24: NFAs, Regular expressions, and NFA→DFA

Highlights

- FSMs with output at states
- State minimization



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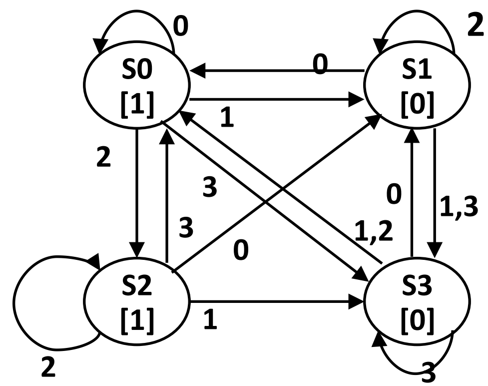
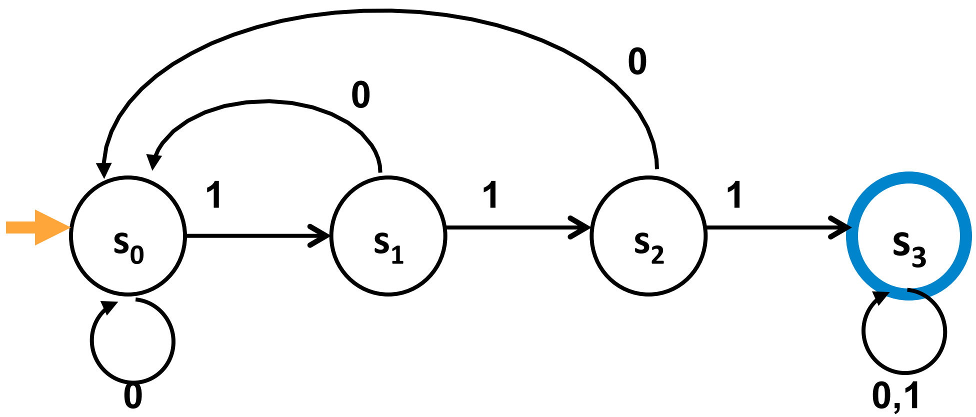


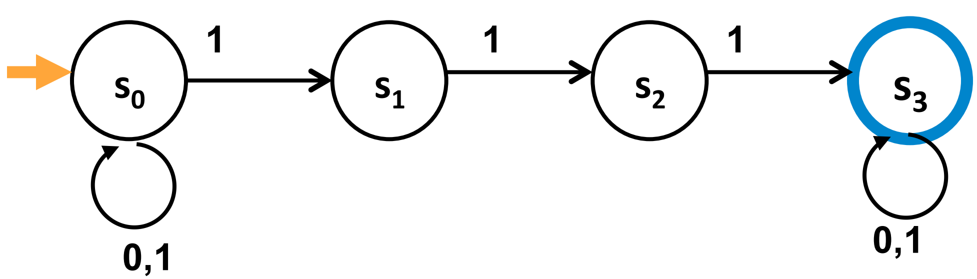
Highlights

Lemma: The language recognized by a DFA is the set of strings x that label some path from its start state to one of its final states



Nondeterministic Finite Automaton (NFA)

- Graph with start state, final states, edges labeled by symbols (like DFA) but
  - Not required to have exactly 1 edge out of each state labeled by each symbol— can have 0 or >1
  - Also can have edges labeled by empty string ε
- **Definition:** x is in the language recognized by an NFA if and only if x labels a path from the start state to some final state



Three ways of thinking about NFAs

- Outside observer: Is there a path labeled by x from the start state to some final state?
- Perfect guesser: The NFA has input x and whenever there is a choice of what to do it magically guesses a good one (if one exists)
- Parallel exploration: The NFA computation runs all possible computations on x step-by-step at the same time in parallel

Goal: NFA to recognize…

Binary strings with even # of 1's or contain the substring 111

NFAs and regular expressions

Theorem: For any set of strings (language) A described by a regular expression, there is an NFA that recognizes A.

Proof idea: Structural induction based on the recursive definition of regular expressions...

Regular Expressions over Σ

- **Basis:**
  - $\varnothing$, ε are regular expressions
  - ***a*** is a regular expression for any $a \in \Sigma$
- **Recursive step:**
  - If **A** and **B** are regular expressions then so are:

    (A ∪ B)

    (AB)

    A*

Base Case

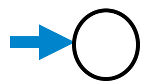
- Case $\emptyset$:

- Case ε:

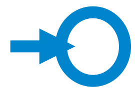
- Case *a*:

Base Case

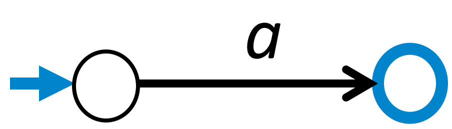
- **Case ∅:**



- **Case ε:**

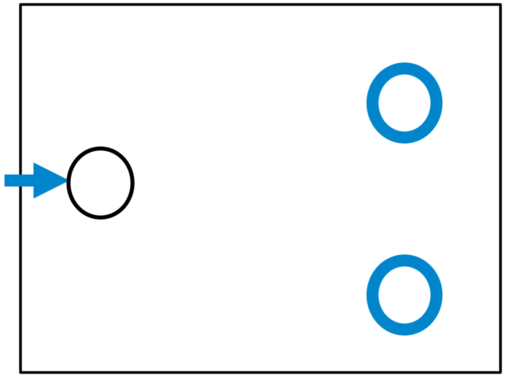


- **Case *a*:**

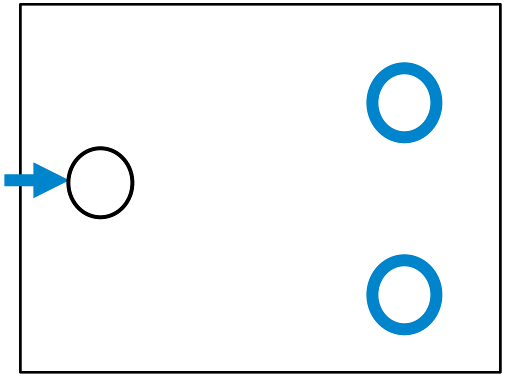


Inductive Hypothesis

- Suppose that for some regular expressions A and B there exist NFAs N_A and N_B such that N_A recognizes the language given by A and N_B recognizes the language given by B



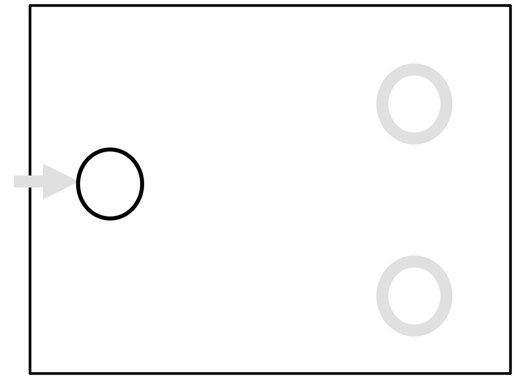
N_A



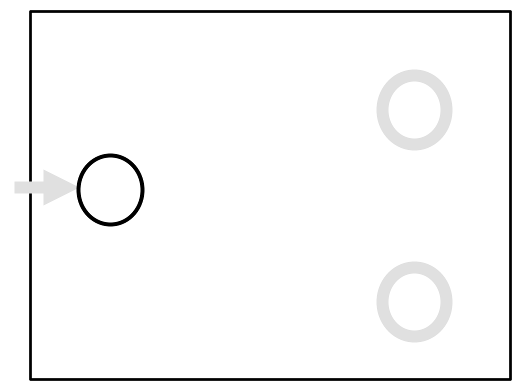
N_B

Inductive Step

Case ($\mathbf{A} \cup \mathbf{B}$):



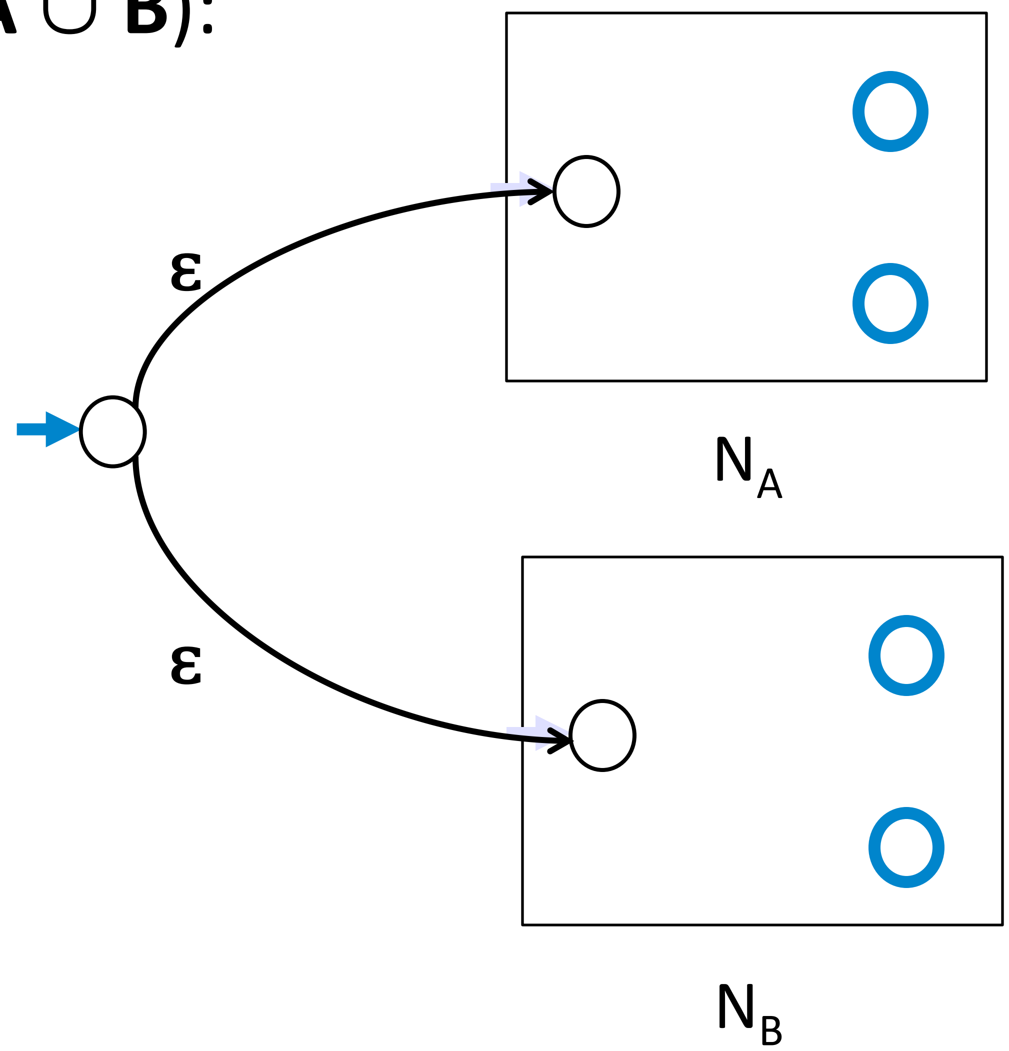
N_A



N_B

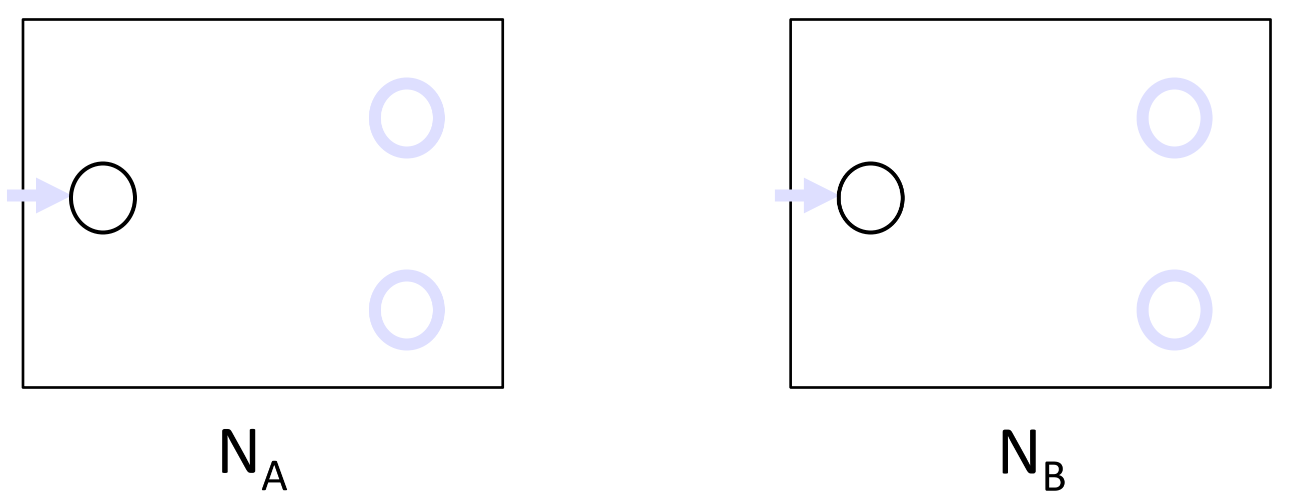
Inductive Step

Case (A ∪ B):



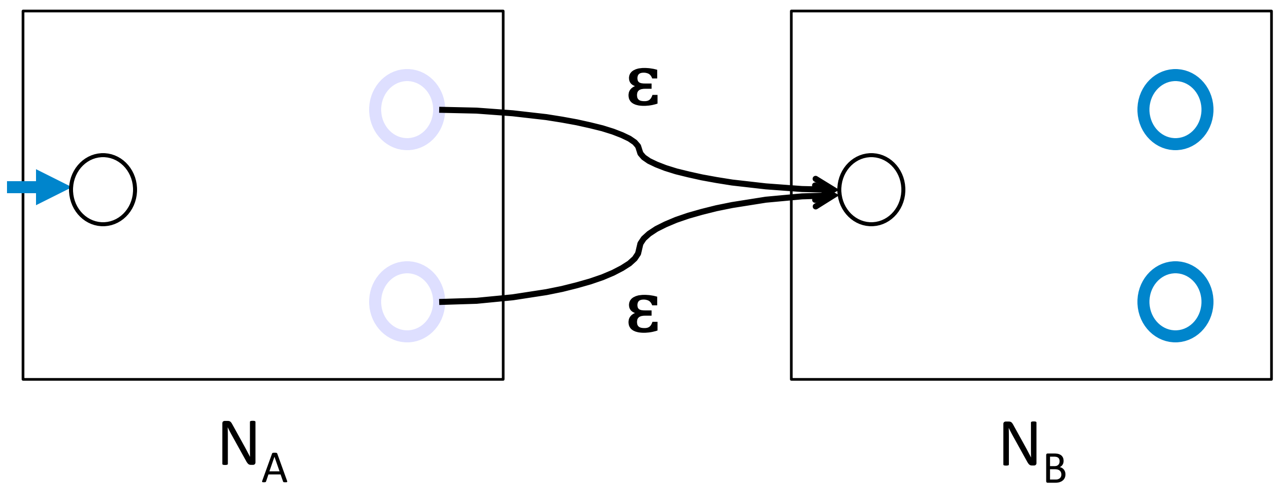
Inductive Step

Case (AB):



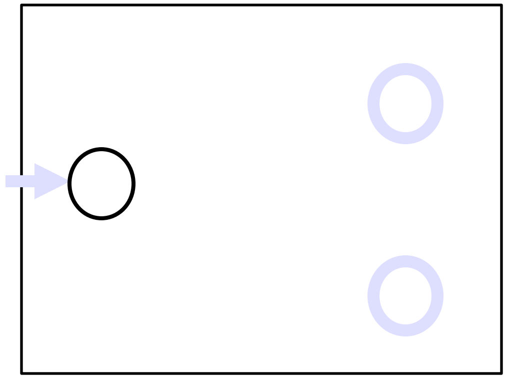
Inductive Step

Case (AB):



Inductive Step

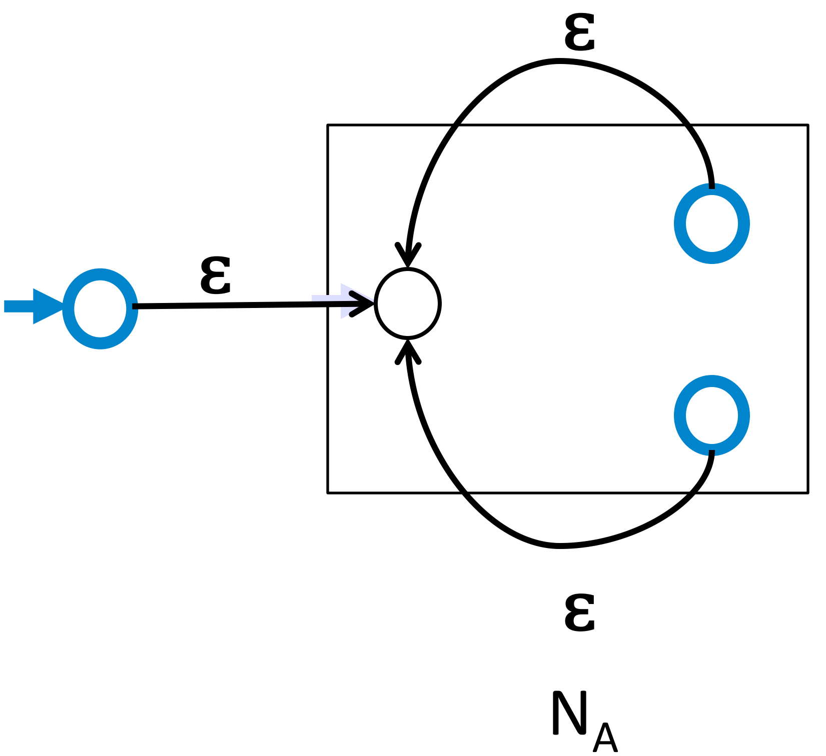
Case A*



N_A

Inductive Step

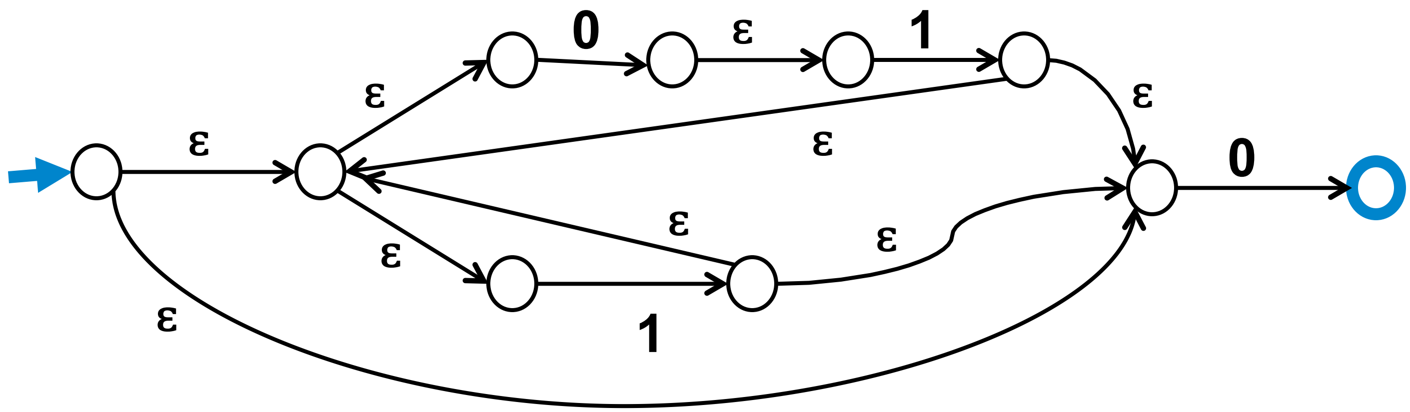
Case A*



Build an NFA for $(01 \cup 1)^*0$

Solution

(01 ∪1)*0



NFAs and DFAs

Every DFA **is** an NFA

- DFAs have requirements that NFAs don't have

Can NFAs recognize more languages?

NFAs and DFAs

Every DFA **is** an NFA

– DFAs have requirements that NFAs don't have

Can NFAs recognize more languages? No!

Theorem: For every NFA there is a DFA that recognizes exactly the same language

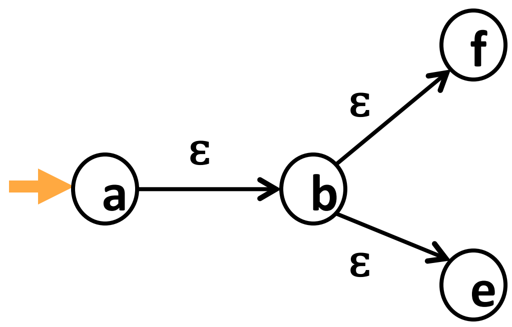
Conversion of NFAs to a DFAs

- Proof Idea:
  - The DFA keeps track of ALL the states that the part of the input string read so far can reach in the NFA
  - There will be one state in the DFA for each *subset* of states of the NFA that can be reached by some string

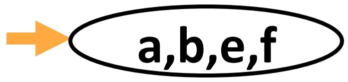
Conversion of NFAs to a DFAs

New start state for DFA

– The set of all states reachable from the start state of the NFA using only edges labeled ε



NFA

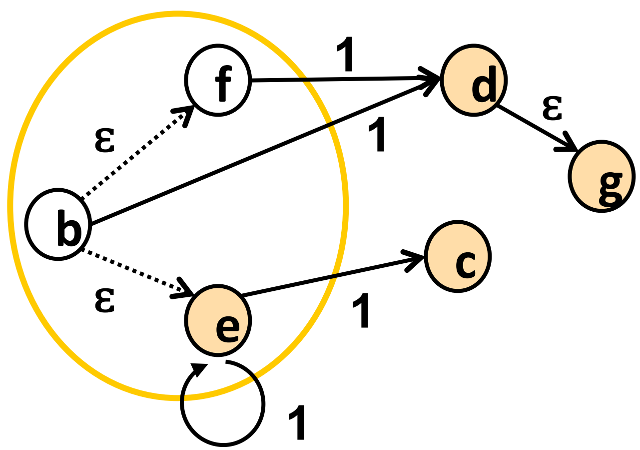


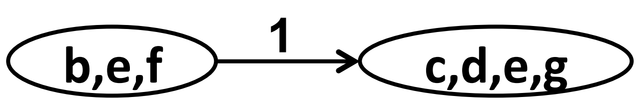
DFA

Conversion of NFAs to a DFAs

For each state of the DFA corresponding to a set S of states of the NFA and each symbol s

- Add an edge labeled **s** to state corresponding to T, the set of states of the NFA reached by
  starting from some state in S, then
  following one edge labeled by **s**, and
  then following some number of edges labeled by **ε**
- T will be ∅ if no edges from S labeled **s** exist

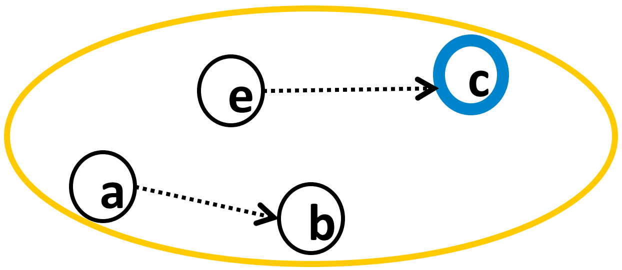




Conversion of NFAs to a DFAs

Final states for the DFA

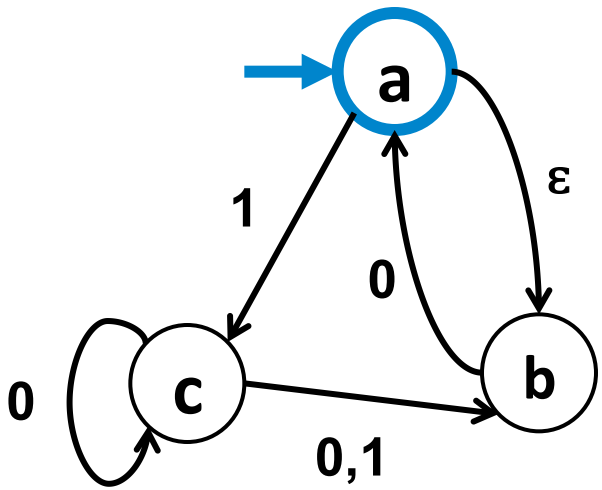
– All states whose set contain some final state of the NFA



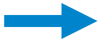
NFA

DFA

Example: NFA to DFA

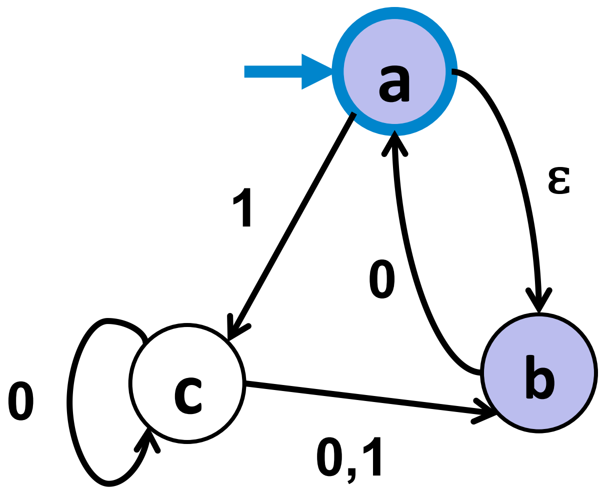


NFA

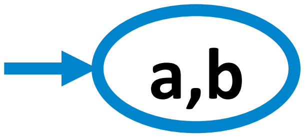


DFA

Example: NFA to DFA

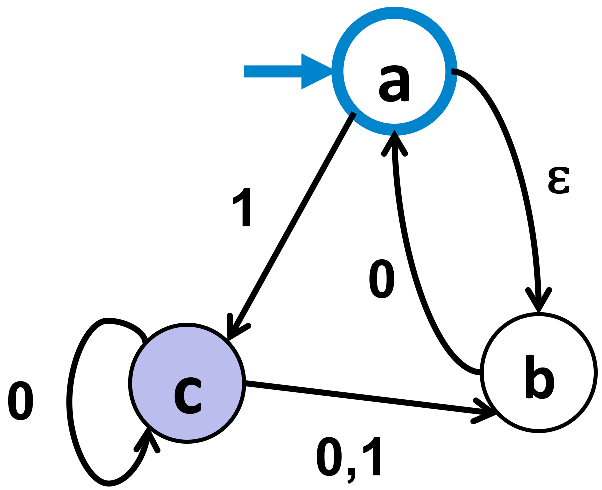


NFA

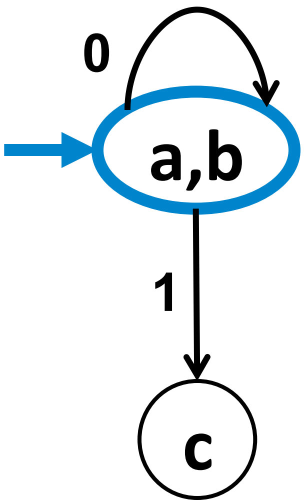


DFA

Example: NFA to DFA

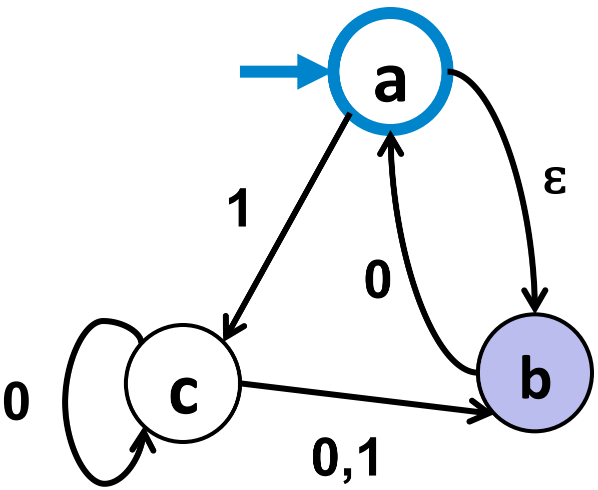


NFA

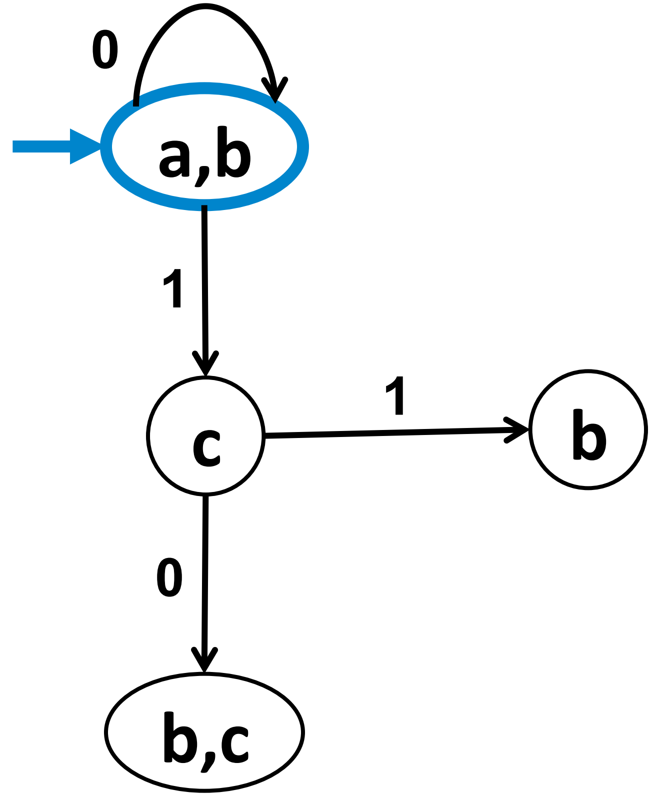


DFA

Example: NFA to DFA

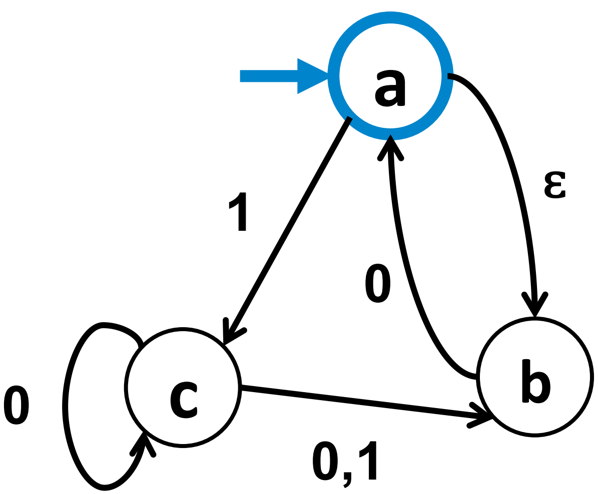


NFA

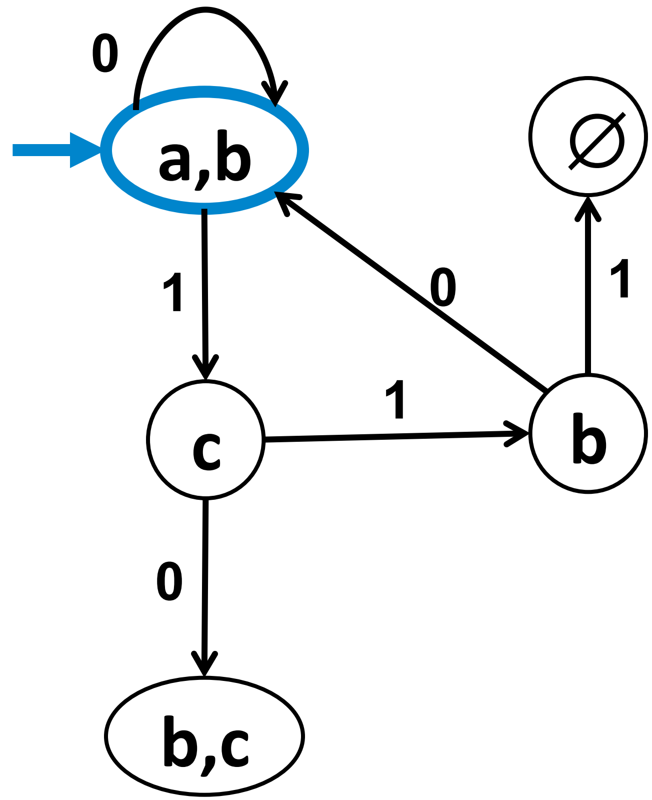


DFA

Example: NFA to DFA

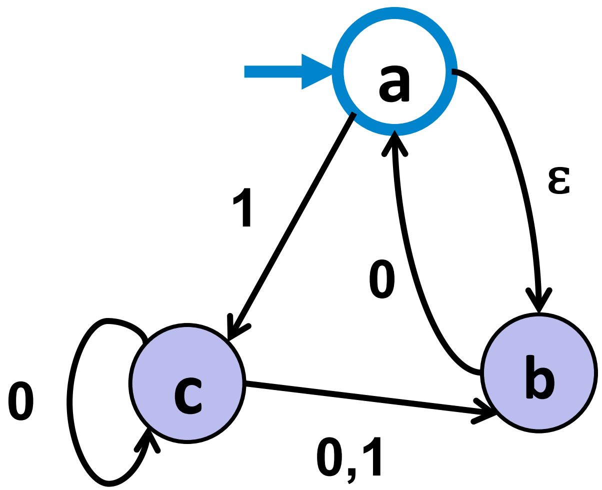


NFA



DFA

Example: NFA to DFA



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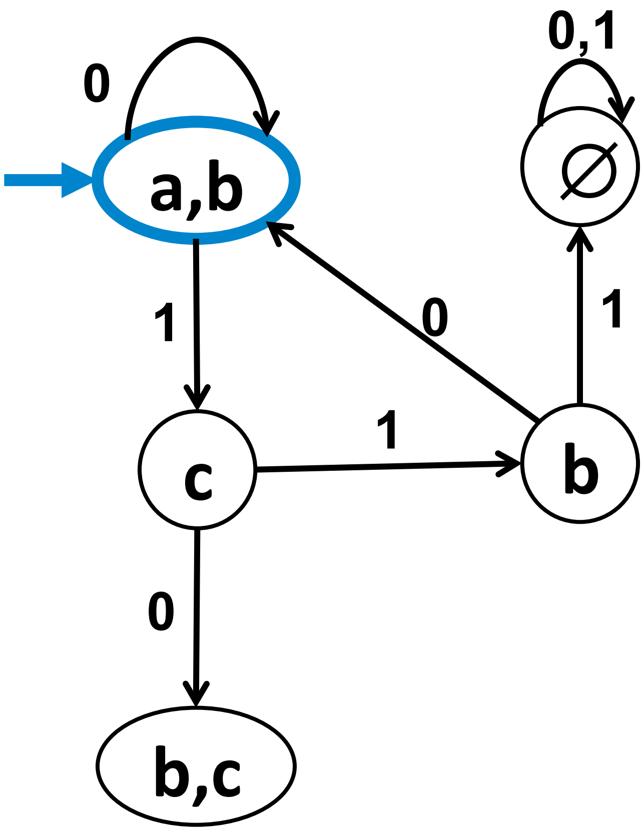
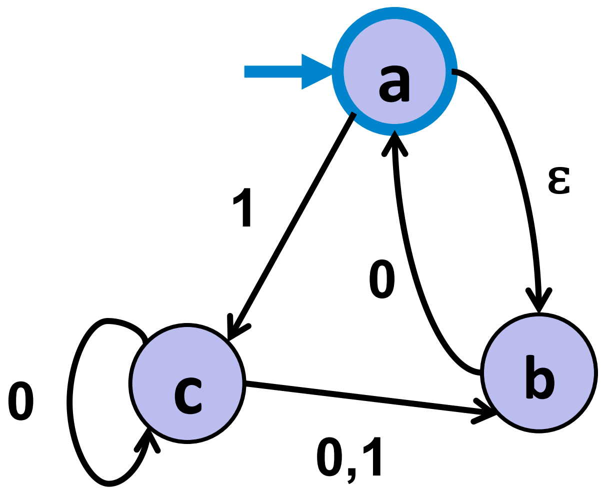
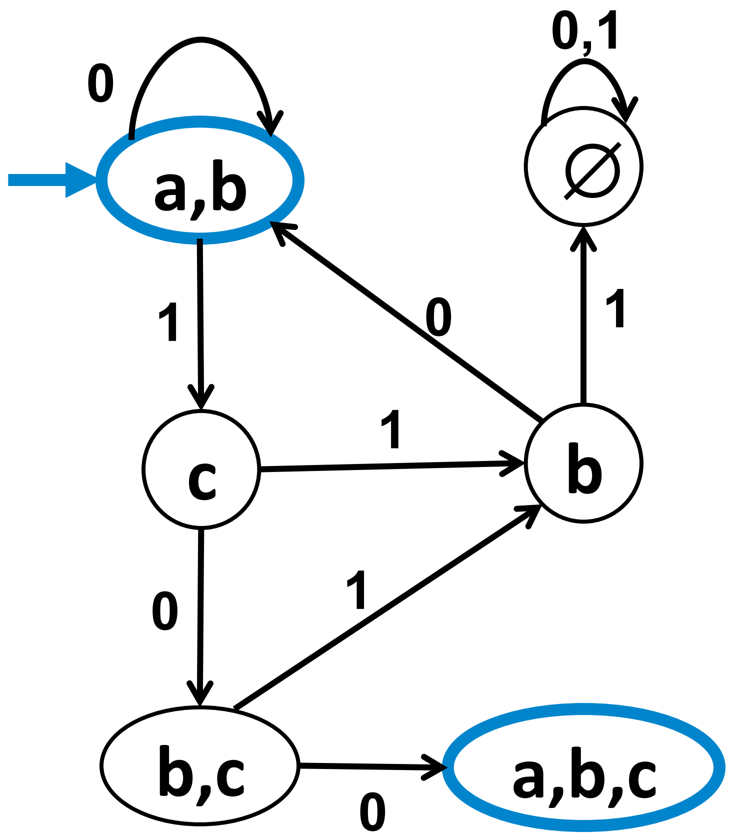


DFA

Example: NFA to DFA

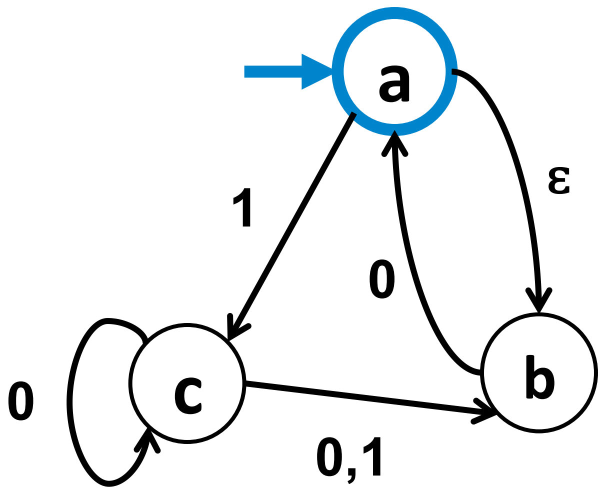


NFA

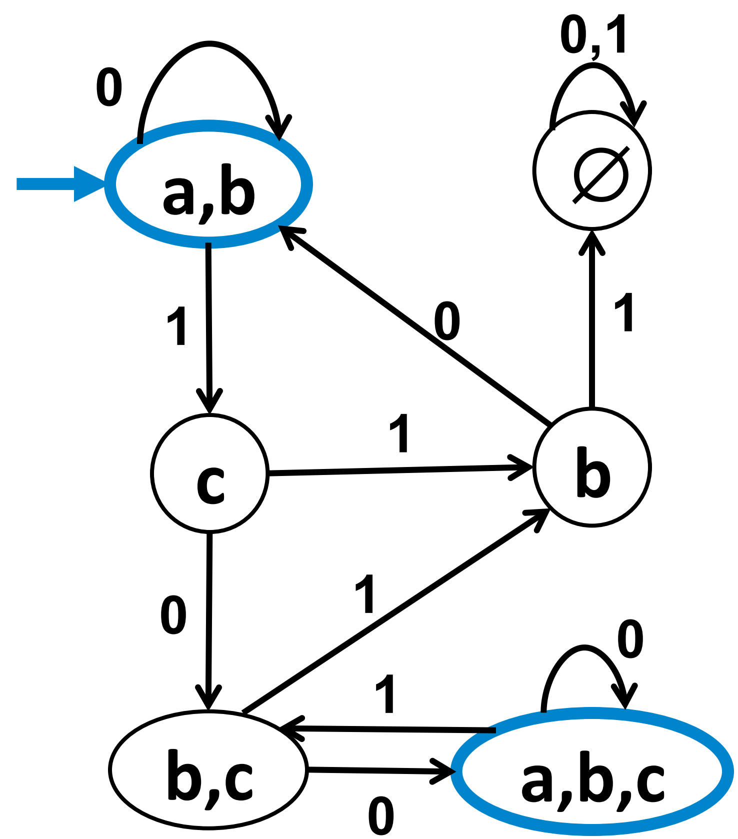


DFA

Example: NFA to DFA



NFA



DFA

Exponential Blow-up in Simulating Nondeterminism

- In general the DFA might need a state for every subset of states of the NFA
  - Power set of the set of states of the NFA
  - n-state NFA yields DFA with at most 2^n states
  - We saw an example where roughly 2^n is necessary

    Is the n^{th} char from the end a 1?

- The famous “P=NP?” question asks whether a similar blow-up is always necessary to get rid of nondeterminism for polynomial-time algorithms