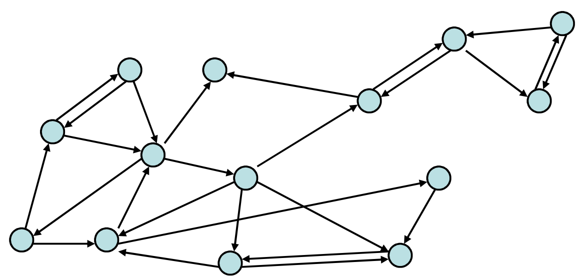
CSE 311: Foundations of Computing

Fall 2014
Lecture 20: Relations and Directed Graphs



Relations

Let A and B be sets,
A binary relation from A to B is a subset of $A \times B$

Let A be a set,
A binary relation on A is a subset of $A \times A$

Relations You Already Know!

$\geq$ on $\mathbb{N}$

That is: $\{(x,y) : x \geq y \text{ and } x, y \in \mathbb{N}\}$

$<$ on $\mathbb{R}$

That is: $\{(x,y) : x < y \text{ and } x, y \in \mathbb{R}\}$

$=$ on Σ^*

That is: $\{(x,y) : x = y \text{ and } x, y \in \Sigma^*\}$

$\subseteq$ on P(U) for universe U

That is: $\{(A,B) : A \subseteq B \text{ and } A, B \in P(U)\}$

Relation Examples

$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$

$R_2 = \{(x, y) \mid x \equiv y \pmod 5\}$

$R_3 = \{(c_1, c_2) \mid c_1 \text{ is a prerequisite of } c_2\}$

$R_4 = \{(s, c) \mid \text{student } s \text{ had taken course } c\}$

Properties of Relations

Let R be a relation on A.

R is reflexive iff $(a,a) \in R$ for every $a \in A$

R is symmetric iff $(a,b) \in R$ implies $(b, a) \in R$

R is antisymmetric iff $(a,b) \in R$ and $a \neq b$ implies $(b,a) \notin R$

R is transitive iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Combining Relations

Let R be a relation from A to B.
Let S be a relation from B to C.

The composition of R and S, $S \circ R$ is the relation from A to C defined by:

$S \circ R = \{(a, c) \mid \exists\, b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$

Intuitively, a pair is in the composition if there is a "connection" from the first to the second.

Examples

$(a,b) \in$ Parent iff b is a parent of a

$(a,b) \in$ Sister iff b is a sister of a

When is $(x,y) \in$ Sister $\circ$ Parent?

When is $(x,y) \in$ Parent $\circ$ Sister?

$S \circ R = \{(a, c) \mid \exists\, b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$

Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother, Husband, Wife express:

Uncle: b is an uncle of a

Cousin: b is a cousin of a

Powers of a Relation

$$R^2 = R \circ R$$
$$= \{(a,c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in R\}$$

$$R^0 = \{(a,a) \mid a \in A\}$$

$$R^1 = R$$

$$R^{n+1} = R^n \circ R$$

Matrix Representation

Relation R **on** $A = \{a_1, \ldots, a_p\}$

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, a_j) \in R, \\ 0 \text{ if } (a_i, a_j) \notin R. \end{cases}$$

{(1, 1), (1, 2), (1, 4), (2,1), (2,3), (3,2), (3, 3), (4,2), (4,3)}

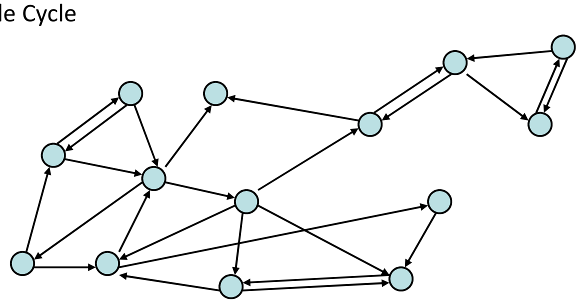
	1	2	3	4
1	1	1	0	1
2	1	0	1	0
3	0	1	1	0
4	0	1	1	0

Directed Graphs

G = (V, E) V – vertices
E – edges, ordered pairs of vertices

Path: $v_0, v_1, \ldots, v_k$, with (v_i, v_{i+1}) in E

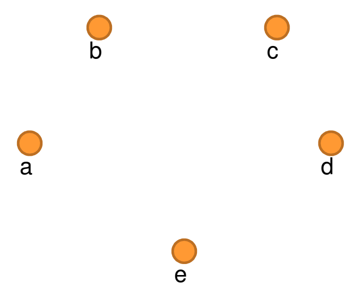
Simple Path
Cycle
Simple Cycle



Representation of Relations

Directed Graph Representation (Digraph)

{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e) (d, e) }



Relational Composition using Digraphs

If $S = \{(2,2), (2,3), (3,1)\}$ **and** $R = \{(1,2), (2,1), (1,3)\}$
Compute $S \circ R$

Paths in Relations and Graphs

A path in a graph of length n is a list of edges with vertices next to each other.

Let R be a relation on a set A. There is a path of length n from a to b if and only if $(a,b) \in R^n$

Connectivity In Graphs

Two vertices in a graph are connected iff there is a path between them.

Let R be a relation on a set A. The connectivity relation R* consists of the pairs (a,b) such that there is a path from a to b in R.

$$R^* = \bigcup_{k=0}^{\infty} R^k$$

Note: The text uses the wrong definition of this quantity.
What the text defines (ignoring k=0) is usually called R^+

Properties of Relations (again)

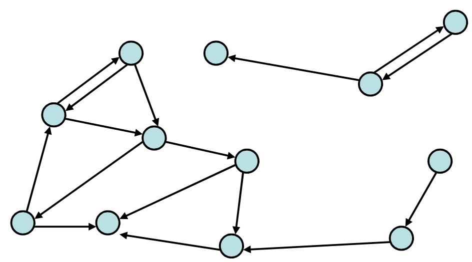
Let R be a relation on A.

R is reflexive iff $(a,a) \in R$ for every $a \in A$

R is symmetric iff $(a,b) \in R$ implies $(b, a) \in R$

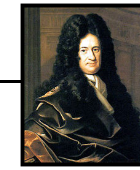
R is transitive iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

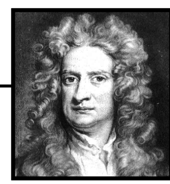
Transitive-Reflexive Closure



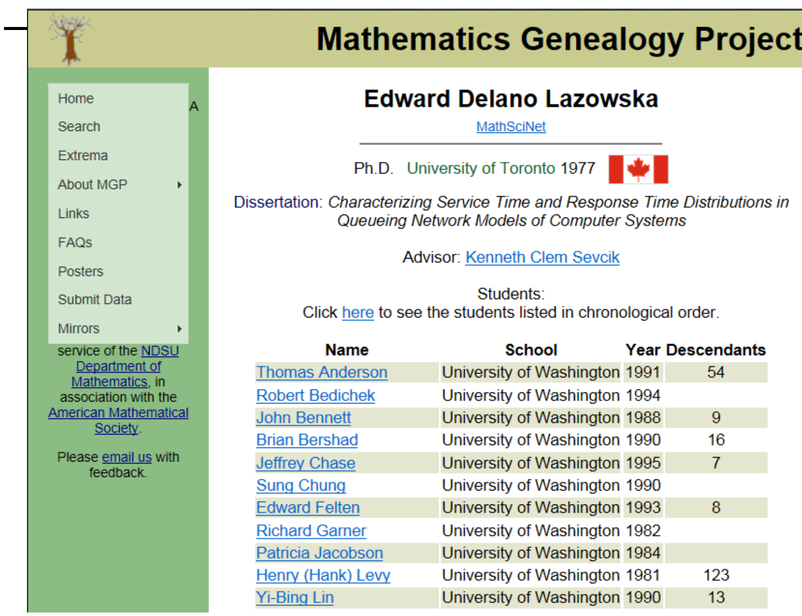
Add the minimum possible number of edges to make the relation transitive and reflexive.

The transitive-reflexive closure of a relation R is the connectivity relation R*

how is  related to  ?

how is  related to  ?

http://genealogy.math.ndsu.nodak.edu/



Mathematics Genealogy Project

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service of the NDSU Department of Mathematics, in association with the American Mathematical Society.

Please email us with feedback.

Edward Delano Lazowska

MathSciNet

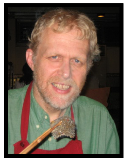
Ph.D. University of Toronto 1977

Dissertation: *Characterizing Service Time and Response Time Distributions in Queueing Network Models of Computer Systems*

Advisor: Kenneth Clem Sevcik

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Thomas Anderson	University of Washington	1991	54
Robert Bedichek	University of Washington	1994	
John Bennett	University of Washington	1988	9
Brian Bershad	University of Washington	1990	16
Jeffrey Chase	University of Washington	1995	7
Sung Chung	University of Washington	1990	
Edward Felten	University of Washington	1993	8
Richard Garner	University of Washington	1982	
Patricia Jacobson	University of Washington	1984	
Henry (Hank) Levy	University of Washington	1981	123
Yi-Bing Lin	University of Washington	1990	13


Anderson

Mayr

Bauer

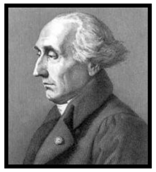
Caratheodory

Minkowski

Klein


Lipschitz

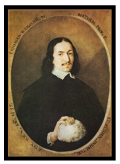
Dirichlet

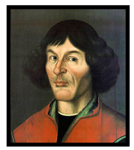
Fourier

Lagrange

Euler

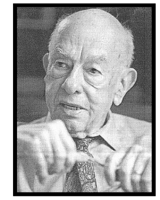
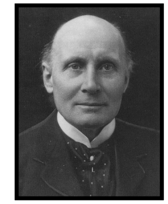
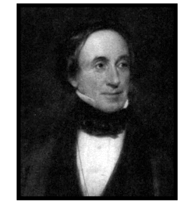

Johann Bernoulli

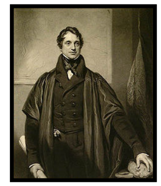
Jacob Bernoulli

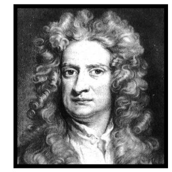
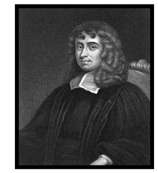
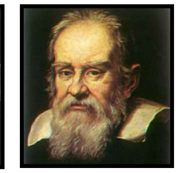
Leibniz

Weigel

Rheticus

Copernicus


Beame

Cook

Quine

Whitehead

Hopkins


Sedgewick

Smith

Newton

Barrow

Galileo

Nicolaus Copernicus
Georg Rheticus
Moritz Steinmetz
Christoph Meurer
Philipp Muller
Erhard Weigel
Gottfried Leibniz
Noclas Malebranache
Jacob Bernoulli
Johann Bernoulli
Leonhard Euler
Joseph Lagrange
Jean-Baptiste Fourier
Gustav Dirichlet
Rudolf Lipschitz
Felix Klein
C. L. Ferdinand Lindemann
Herman Minkowski
Constantin Caratheodory
Georg Aumann
Friedrich Bauer
Manfred Paul
Ernst Mayr
Richard Anderson

Galileo Galilei
Vincenzo Viviani
Issac Barrow
Isaac Newton
Roger Cotes
Robert Smith
Walter Taylor
Stephen Whisson
Thomas Postlethwaite
Thomas Jones
Adam Sedgewick
William Hopkins
Edward Routh
Alfred North Whitehead
Willard Quine
Hao Wang
Stephen Cook
Paul Beame

n-ary Relations

Let $A_1, A_2, \ldots, A_n$ be sets. An **n-ary** relation on these sets is a subset of $A_1 \times A_2 \times \cdots \times A_n$.

Relational Databases

STUDENT

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Von Neuman	481080220	555	3.78
Russell	238082388	022	3.85
Einstein	238001920	022	2.11
Newton	1727017	333	3.61
Karp	348882811	022	3.98
Bernoulli	2921938	022	3.21

Relational Databases

STUDENT

Student_Name	ID_Number	Office	GPA	Course
Knuth	328012098	022	4.00	CSE311
Knuth	328012098	022	4.00	CSE351
Von Neuman	481080220	555	3.78	CSE311
Russell	238082388	022	3.85	CSE312
Russell	238082388	022	3.85	CSE344
Russell	238082388	022	3.85	CSE351
Newton	1727017	333	3.61	CSE312
Karp	348882811	022	3.98	CSE311
Karp	348882811	022	3.98	CSE312
Karp	348882811	022	3.98	CSE344
Karp	348882811	022	3.98	CSE351
Bernoulli	2921938	022	3.21	CSE351

What's not so nice?

relational databases

STUDENT

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Von Neuman	481080220	555	3.78
Russell	238082388	022	3.85
Einstein	238001920	022	2.11
Newton	1727017	333	3.61
Karp	348882811	022	3.98
Bernoulli	2921938	022	3.21

TAKES

ID_Number	Course
328012098	CSE311
328012098	CSE351
481080220	CSE311
238082388	CSE312
238082388	CSE344
238082388	CSE351
1727017	CSE312
348882811	CSE311
348882811	CSE312
348882811	CSE344
348882811	CSE351
2921938	CSE351

Better

database operations: projection

Find all offices: $\Pi_{\text{Office}}(\text{STUDENT})$

Office
022
555
333

Find offices and GPAs: $\Pi_{\text{Office,GPA}}(\text{STUDENT})$

Office	GPA
022	4.00
555	3.78
022	3.85
022	2.11
333	3.61
022	3.98
022	3.21

database operations: selection

Find students with GPA > 3.9 : $\sigma_{GPA>3.9}$(STUDENT)

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Karp	348882811	022	3.98

Retrieve the name and GPA for students with GPA > 3.9:
$\Pi_{Student\_Name,GPA}(\sigma_{GPA>3.9}(STUDENT))$

Student_Name	GPA
Knuth	4.00
Karp	3.98

database operations: natural join

Student ⋈ Takes

Student_Name	ID_Number	Office	GPA	Course
Knuth	328012098	022	4.00	CSE311
Knuth	328012098	022	4.00	CSE351
Von Neuman	481080220	555	3.78	CSE311
Russell	238082388	022	3.85	CSE312
Russell	238082388	022	3.85	CSE344
Russell	238082388	022	3.85	CSE351
Newton	1727017	333	3.61	CSE312
Karp	348882811	022	3.98	CSE311
Karp	348882811	022	3.98	CSE312
Karp	348882811	022	3.98	CSE344
Karp	348882811	022	3.98	CSE351
Bernoulli	2921938	022	3.21	CSE351