## CSE 311: Foundations of Computing

Fall 2014

Lecture 19: Regular Expressions
Context-Free Grammars


## Review: each regular expression is a "pattern"

ε matches the **empty string**

***a*** matches the one character string *a*

**(A ∪ B)** matches all strings that either **A** matches or **B** matches (or both)

**(AB)** matches all strings that have a first part that **A** matches followed by a second part that **B** matches

**A*** matches all strings that have any number of strings (even 0) that **A** matches, one after another

## Examples

- All binary strings that have an even # of 1's

- All binary strings that *don't* contain 101

## Limitations of Regular Expressions

- Not all languages can be specified by regular expressions
- Even some easy things like
  - Palindromes
  - Strings with equal number of 0's and 1's
- But also more complicated structures in programming languages
  - Matched parentheses
  - Properly formed arithmetic expressions
  - etc.

## Context-Free Grammars

- A Context-Free Grammar (CFG) is given by a finite set of substitution rules involving
  - A finite set **V** of *variables* that can be replaced
  - Alphabet $\Sigma$ of *terminal symbols* that can't be replaced
  - One variable, usually **S**, is called the *start symbol*
- The rules involving a variable **A** are written as

  $$\mathbf{A} \to w_1 \mid w_2 \mid \cdots \mid w_k$$

  where each $w_i$ is a string of variables and terminals – that is $w_i \in (\mathbf{V} \cup \Sigma)^*$

## How CFGs generate strings

- Begin with start symbol **S**
- If there is some variable **A** in the current string you can replace it by one of the w's in the rules for **A**
  - $\mathbf{A} \to w_1 \mid w_2 \mid \cdots \mid w_k$
  - Write this as $x\mathbf{A}y \Rightarrow xwy$
  - Repeat until no variables left
- The set of strings the CFG generates are all strings produced in this way that have no variables

## Example Context-Free Grammars

**Example:** $\mathbf{S} \to 0\mathbf{S}0 \mid 1\mathbf{S}1 \mid 0 \mid 1 \mid \varepsilon$

**Example:** $\mathbf{S} \to 0\mathbf{S} \mid \mathbf{S}1 \mid \varepsilon$

## Example Context-Free Grammars

**Grammar for $\{0^n 1^n : n \geq 0\}$**

**(all strings with same # of 0's and 1's with all 0's before 1's)**

**Example:** $\mathbf{S} \to (\mathbf{S}) \mid \mathbf{SS} \mid \varepsilon$

## Simple Arithmetic Expressions

**E→ E+E | E∗E | (E) | x | y | z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9**

Generate (2∗x) + y

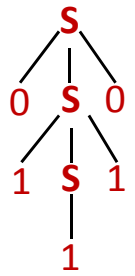
Generate x+y∗z in two fundamentally different ways

## Parse Trees

Suppose that grammar G generates a string x

- A *parse tree* of x for G has
  - Root labeled S (start symbol of G)
  - The children of any node labeled A are labeled by symbols of w left-to-right for some rule A → w
  - The symbols of x label the leaves ordered left-to-right

S → 0S0 | 1S1 | 0 | 1 | ε




Parse tree of 01110

## CFGs and recursively-defined sets of strings

- A CFG with the start symbol **S** as its only variable recursively defines the set of strings of terminals that **S** can generate
- A CFG with more than one variable is a simultaneous recursive definition of the sets of strings generated by *each* of its variables
  - Sometimes necessary to use more than one

## building precedence in simple arithmetic expressions

- **E** – expression (start symbol)
- **T** – term **F** – factor **I** – identifier **N** - number

**E** → **T** | **E**+**T**

**T** → **F** | **F**∗**T**

**F** → (**E**) | **I** | **N**

**I** → x | y | z

**N** → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

## Backus-Naur Form (The same thing...)

BNF (Backus-Naur Form) grammars

- Originally used to define programming languages
- Variables denoted by long names in angle brackets, e.g.

  <identifier>, <if-then-else-statement>, <assignment-statement>, <condition>

  ::= used instead of →

## BNF for C

```
statement:
  ((identifier | "case" constant-expression | "default") ":")*
  (expression? ";" |
   block |
   "if" "(" expression ")" statement |
   "if" "(" expression ")" statement "else" statement |
   "switch" "(" expression ")" statement |
   "while" "(" expression ")" statement |
   "do" statement "while" "(" expression ")" ";" |
   "for" "(" expression? ";" expression? ";" expression? ")" statement |
   "goto" identifier ";" |
   "continue" ";" |
   "break" ";" |
   "return" expression? ";"
  )

block: "{" declaration* statement* "}"

expression:
  assignment-expression%

assignment-expression: (
    unary-expression (
      "=" | "*=" | "/=" | "%=" | "+=" | "-=" | "<<=" | ">>=" | "&=" |
      "^=" | "|="
    )
  )* conditional-expression

conditional-expression:
  logical-OR-expression ( "?" expression ":" conditional-expression )?
```

## Parse Trees

Back to middle school:

<sentence>::=<noun phrase><verb phrase>

<noun phrase>::==<article><adjective><noun>

<verb phrase>::=<verb><adverb>|<verb><object>

<object>::=<noun phrase>

Parse:

The yellow duck squeaked loudly

The red truck hit a parked car