CSE 311: Foundations of Computing

Fall 2014

Lecture 18: Structural induction, regular expressions

Administrivia

Midterm back today!

Average 80%

Median 81%

Range	Number
<60	
60 – 69	
70 – 79	
80 – 89	
90+	

Graded Homework 5 back Friday

Homework 6 out later today

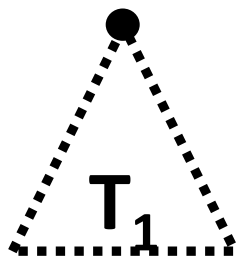
Structural Induction

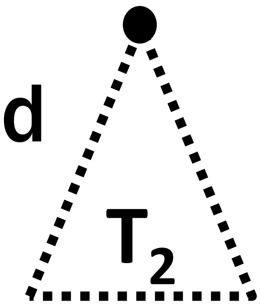
How to prove $\forall\, x \in S, P(x)$ is true:

- **Base Case:** Show that P(u) is true for all specific elements of $u \in S$ mentioned in the *Basis step*
- **Inductive Hypothesis:** Assume that P is true for some arbitrary values of each of the existing named elements mentioned in the *Recursive step*
- **Inductive Step:** Prove that P(w) holds for each of the new elements constructed in the *Recursive step* using the named elements mentioned in the Inductive Hypothesis
- **Conclude** that $\forall\, x \in S, P(x)$

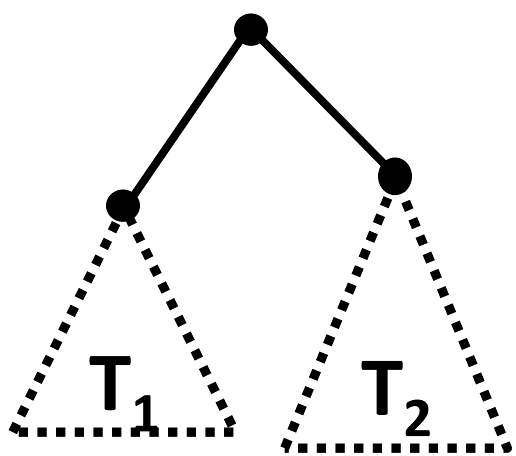
Rooted Binary Trees

- **Basis:**  is a rooted binary tree

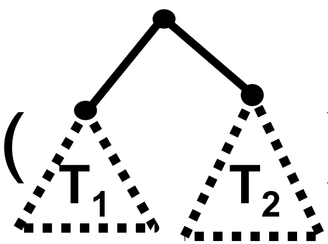
- **Recursive step:** If 

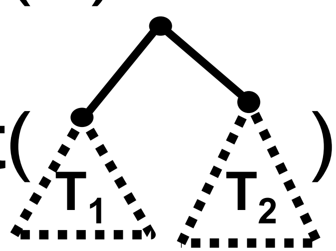
and 

are rooted binary trees

  then so is: 

Functions Defined on Rooted Binary Trees

- size(•)=1
- size(

) = 1+size(T_1)+size(T_2)
- height(•)=0
- height(

)=1+max{height(T_1),height(T_2)}

Claim: For every rooted binary tree T, $\text{size}(T) \le 2^{\text{height}(T)+1} - 1$

Let $P(T)$ be "$\text{size}(T) \le 2^{\text{height}(T)+1} - 1$". We go by structural induction on the definition of trees.

Base Case: When $T = \bullet$, $\text{size}(T) = 1$,

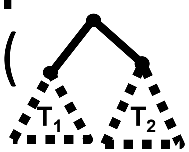
$\text{height}(T) = 0$,

Note that $1 \le 2^1 - 1 = 2^{0+1} - 1$.

So, $P(\bullet)$ is true.

Induction Hypothesis: Suppose $P(T_1)$ and $P(T_2)$ are true for some T_1, T_2.

Induction Step:

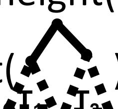
Note that size(

$) = 1 + \text{size}(T_1) + \text{size}(T_2)$

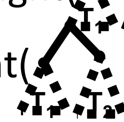
$\le 1 + 2^{\text{height}(T1)+1} - 1 + 2^{\text{height}(T2)+1} - 1$ (by IH)

$\le 2^{\text{height}(T1)+1} + 2^{\text{height}(T2)+1} - 1$

$\le 2(2^{\max(\text{height}(T1),\text{height}(T2))+1}) - 1$

$\le 2(2^{\text{height}($ 

$)}) - 1$

$\le 2^{\text{height}($ 

$)+1}) - 1$

So, the $P(T)$ is true for all trees by structural induction.

Languages: Sets of Strings

- Sets of strings that satisfy special properties are called *languages*. Examples:
  - English sentences
  - Syntactically correct Java/C/C++ programs
  - Σ^* = All strings over alphabet Σ
  - Palindromes over Σ
  - Binary strings that don't have a 0 after a 1
  - Legal variable names. keywords in Java/C/C++
  - Binary strings with an equal # of 0's and 1's

Regular Expressions

Regular expressions over Σ

- **Basis:**

  $\varnothing$, $\boldsymbol{\varepsilon}$ are regular expressions

  a is a regular expression for any $a \in \Sigma$

- **Recursive step:**
  - If **A** and **B** are regular expressions then so are:

    (A $\cup$ B)

    (AB)

    A*

Each Regular Expression is a "pattern"

ε matches the **empty string**

a matches the one character string *a*

(A ∪ B) matches all strings that either **A** matches or **B** matches (or both)

(AB) matches all strings that have a first part that **A** matches followed by a second part that **B** matches

A* matches all strings that have any number of strings (even 0) that **A** matches, one after another

Examples

- 001^*
- 0^*1^*
- $(0 \cup 1)0(0 \cup 1)0$
- $(0^*1^*)^*$
- $(0 \cup 1)^*\, 0110\, (0 \cup 1)^*$
- $(00 \cup 11)^*\, (01010 \cup 10001)(0 \cup 1)^*$

Regular Expressions in Practice

- Used to define the "tokens": e.g., legal variable names, keywords in programming languages and compilers
- Used in **`grep`**, a program that does pattern matching searches in UNIX/LINUX
- Pattern matching using regular expressions is an essential feature of PHP
- We can use regular expressions in programs to process strings!

Regular Expressions in Java

- Pattern p = Pattern.compile("a*b");
- Matcher m = p.matcher("aaaaab");
- boolean b = m.matches();

  `[01]` a 0 or a 1 **^** start of string **$** end of string

  `[0-9]` any single digit `\.` period `\,` comma `\-` minus

  `.` any single character

  ab a followed by b (**AB**)

  (a|b) a or b (**A** $\cup$ **B**)

  a**?** zero or one of a (**A** $\cup$ λ)

  a***** zero or more of a **A***

  a**+** one or more of a **AA***

- e.g. `^[\-+]?[0-9]*(\.|\,)?[0-9]+$`

  General form of decimal number e.g. 9.12 or -9,8 (Europe)