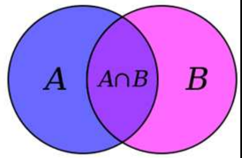
CSE 311



Foundations of Computing I

Fall 2014

It's Boolean algebra again

- Definition for $\cup$ based on $\vee$

$$A \cup B = \{\, x : (x \in A) \vee (x \in B\,)\}$$

- Definition for $\cap$ based on $\wedge$

$$A \cap B = \{\, x : (x \in A) \wedge (x \in B)\}$$

- Complement works like $\neg$

$$\bar{A} = \{\, x : x \notin A\,\}$$

(with respect to universe U)

De Morgan's Laws

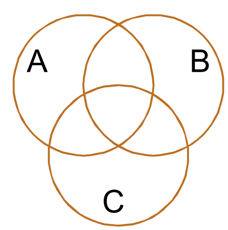
$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

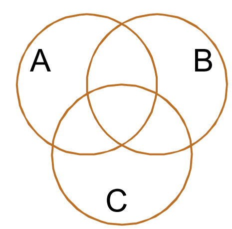
Let U be the universe.

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$





Representing Sets Using Bits

- Suppose universe U is $\{1,2,\ldots,n\}$
- Can represent set $B \subseteq U$ as a vector of bits:

  $b_1 b_2 \ldots b_n$ where $b_i = 1$ when $i \in B$

  $b_i = 0$ when $i \notin B$

  – Called the *characteristic vector* of set B

- Given characteristic vectors for A and B
  – What is characteristic vector for $A \cup B$? $A \cap B$?

UNIX/Linux File Permissions

- `ls -l`

```
drwxr-xr-x ... Documents/
-rw-r--r-- ... file1
```

- Permissions maintained as bit vectors
  – Letter means bit is 1
  – "–" means bit is 0.

Bitwise Operations

$$\begin{array}{rl} & 01101101 \\ \vee & \underline{00110111} \\ & 01111111 \end{array}$$

Java: `z=x|y`

$$\begin{array}{rl} & 00101010 \\ \wedge & \underline{00001111} \\ & 00001010 \end{array}$$

Java: `z=x&y`

$$\begin{array}{rl} & 01101101 \\ \oplus & \underline{00110111} \\ & 01011010 \end{array}$$

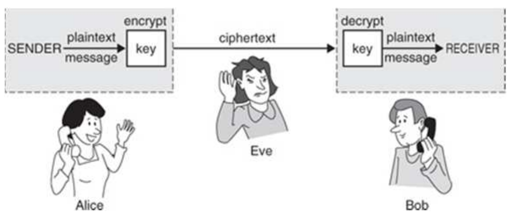
Java: `z=x^y`

A Useful Identity

- If x and y are bits: $(x \oplus y) \oplus y = ?$

Private Key Cryptography

- Alice wants to communicate message secretly to Bob so that eavesdropper Eve who hears their conversation cannot tell what Alice's message is.
- Alice and Bob can get together and privately share a secret key K ahead of time.



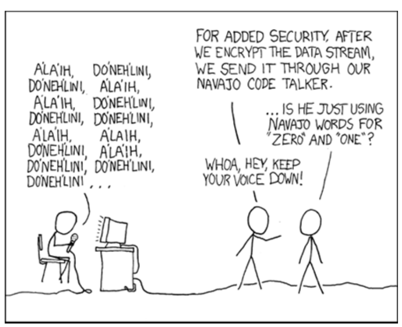
One-Time Pad

- Alice and Bob privately share random n-bit vector K
  - Eve does not know K
- Later, Alice has n-bit message m to send to Bob
  - Alice computes $C = m \oplus K$
  - Alice sends C to Bob
  - Bob computes $m = C \oplus K$ which is $(m \oplus K) \oplus K$
- Eve cannot figure out m from C unless she can guess K

CSE 311: Foundations of Computing

Fall 2014

Lecture 10: Functions, Modular arithmetic



Announcements

Homework 3 due now

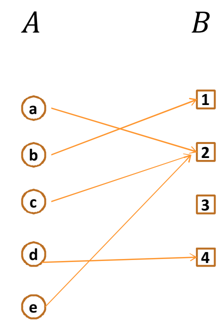
Homework 2 Solutions available

Homework 4 out later today

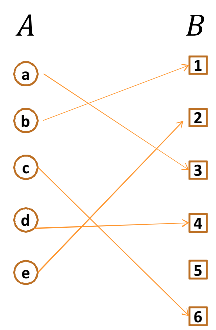
Functions

- A *function* from A to B.
  - Every element of A is assigned to exactly one element of B.
  - We write $f: A \rightarrow B$.
  - "Image of X" = $\{x : \exists y\ (y \in X \wedge x = f(y))\}$
- *Domain* of *f* is A
- Codomain of f is B
- *Image* of f = Image of domain = all the elements pointed to by something in the domain.

Image



Is this a function? One-to-One? Onto?



Functional Examples

Domain: Reals

	One-to-one	Onto
$x \mapsto x^2$		
$x \mapsto x^3 - x$		
$x \mapsto e^x$		
$x \mapsto x^3$		

Number Theory (and applications to computing)

- Branch of Mathematics with direct relevance to computing
- Many significant applications
  - Cryptography
  - Hashing
  - Security
- Important tool set

Modular Arithmetic

- Arithmetic over a finite domain
- In computing, almost all computations are over a finite domain

I'm ALIVE!

```
public class Test {
   final static int SEC_IN_YEAR = 364*24*60*60*100;
   public static void main(String args[]) {
      System.out.println(
         "I will be alive for at least " +
         SEC_IN_YEAR * 101 + " seconds."
      );
   }
}
```

```
 ----jGRASP exec: java Test
I will be alive for at least -186619904 seconds.

 ----jGRASP: operation complete.
```

Divisibility

Integers a, b, with a ≠ 0, we say that a *divides* b iff there is an integer k such that b = ka. The notation a | b denotes "a divides b."

Division Theorem

Let a be an integer and d a positive integer. Then there are *unique* integers q and r, with $0 \le r < d$, such that $a = dq + r$.

$q = a$ **div** d $\quad$ $r = a$ **mod** d

Note: $r \ge 0$ even if $a < 0$.
Not quite the same as `a % d`.

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```
public class Test2 {
    public static void main(String args[]) {
        int a = -5;
        int d = 2;
        System.out.println(a % d);
    }
}
```

```
----jGRASP exec: java Test2
-1
----jGRASP: operation complete.
```

Note: $r \ge 0$ even if $a < 0$.
Not quite the same as `a % d`.

Arithmetic, mod 7

$$a +_7 b = (a + b) \bmod 7$$

$$a \times_7 b = (a \times b) \bmod 7$$

+	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

x	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

Modular Arithmetic

Let a and b be integers, and m be a positive integer. We say a *is congruent to b modulo m* if m divides a – b. We use the notation $a \equiv b \pmod{m}$ to indicate that a is congruent to b modulo m.

Modular Arithmetic: Examples

A ≡ 0 (mod 2)

This statement is the same as saying "A is even"; so, any A that is even (including negative even numbers) will work.

1 ≡ 0 (mod 4)

This statement is false. If we take it mod 1 instead, then the statement is true.

A ≡ -1 (mod 17)

If A = 17x – 1 = 17x + 16, then it works.

Note that (m – 1) mod m
= ((m mod m) + (-1 mod m)) mod m
= (0 + -1) mod m
= -1 mod m

Modular Arithmetic: A Property

Let a and b be integers, and let m be a positive integer. Then a ≡ b (mod m) if and only if a mod m = b mod m.