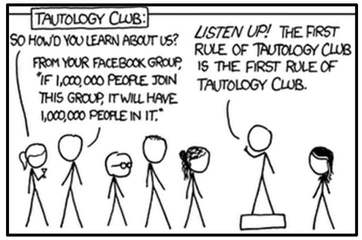
CSE 311



Foundations of Computing I

Fall 2014

Announcements

Homework #2 due today

- Solutions available (paper format) in front
- HW #3 will be posted tonight

Inference Rules

- Each **inference rule** is written as: ...which means that if both A and B are true then you can infer C and you can infer D.
  - For rule to be correct $(A \wedge B) \rightarrow C$ and $(A \wedge B) \rightarrow D$ must be a tautologies

$$\frac{A,\ B}{\therefore C, D}$$

- Sometimes rules don't need anything to start with. These rules are called **axioms**:
  - e.g. *Excluded Middle Axiom*

$$\frac{}{\therefore\ p \vee \neg p}$$

Simple Propositional Inference Rules

Excluded middle plus two inference rules per binary connective, one to eliminate it and one to introduce it

	Elim	Intro
$\wedge$	$\frac{p \wedge q}{\therefore p, q}$	$\frac{p, q}{\therefore p \wedge q}$
$\vee$	$\frac{p \vee q,\ \neg p}{\therefore q}$	$\frac{p}{\therefore p \vee q,\ q \vee p}$
	$\frac{p,\ p \rightarrow q}{\therefore q}$	$\frac{p \Rightarrow q}{\therefore p \rightarrow q}$ Direct Proof Rule — Not like other rules

Important: Application of Inference Rules

- You can use equivalences to make substitutions of any sub-formula.
- Inference rules only can be applied to whole formulas (not correct otherwise).

e.g. 1. $p \rightarrow q$ given

~~2. $(p \vee r) \rightarrow q$ intro $\vee$ from 1.~~

Does not follow! e.g . p=F, q=F, r=T

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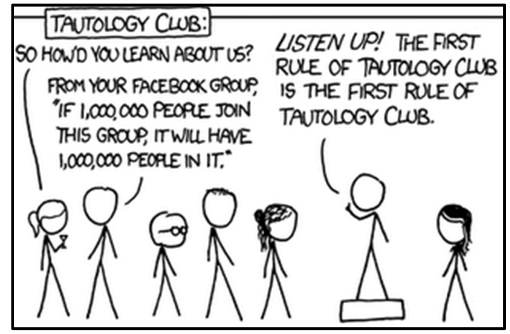
Lecture 7: Proofs



Proofs

Show that $\neg r$ follows from $p \wedge s$, $q \rightarrow \neg r$, and $s \vee \neg q$.

Direct Proof of an Implication

- $p \Rightarrow q$ denotes a proof of q given p as an assumption
- The direct proof rule:
  If you have such a proof then you can conclude that $p \rightarrow q$ is true

proof subroutine

Example:

1. p assumption
2. $p \vee q$ intro for $\vee$ from 1
3. $p \rightarrow (p \vee q)$ direct proof rule

Proofs using the direct proof rule

Show that $p \rightarrow r$ follows from q and $(p \wedge q) \rightarrow r$

1. q — given
2. $(p \wedge q) \rightarrow r$ — given
    3. p — assumption
    4. $p \wedge q$ — from 1 and 3 via Intro $\wedge$ rule
    5. r — modus ponens from 2 and 4
6. $p \rightarrow r$ — direct proof rule

Example

Prove: $(p \wedge q) \rightarrow (p \vee q)$

Example

Prove: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

One General Proof Strategy

1. Look at the rules for introducing connectives to see how you would build up the formula you want to prove from pieces of what is given
2. Use the rules for eliminating connectives to break down the given formulas so that you get the pieces you need to do 1.
3. Write the proof beginning with what you figured out for 2 followed by 1.

Inference rules for quantifiers

$$\frac{P(c) \text{ for some } c}{\therefore \exists x\, P(x)} \qquad \frac{\forall x\, P(x)}{\therefore P(a) \text{ for any } a}$$

$$\frac{\text{"Let } a \text{ be anything*"} \ldots P(a)}{\therefore \forall x\, P(x)} \qquad \frac{\exists x\, P(x)}{\therefore P(c) \text{ for some } special\text{** } c}$$

* in the domain of P

** By special, we mean that c is a name for a value where P(c) is true. We can't use anything else about that value, so c has to be a NEW variable!

Proofs using Quantifiers

"There exists an even prime number"

Prime(*x*): x is an integer > 1 and x is not a multiple of any integer strictly between 1 and x

Even and Odd

Even(x) $\equiv \exists y\ (x=2y)$
Odd(x) $\equiv \exists y\ (x=2y+1)$
Domain: Integers

Prove: "The square of every even number is even."

Formal proof of: $\forall x\,(\text{Even}(x) \rightarrow \text{Even}(x^2))$

Even and Odd

Even(x) $\equiv \exists y\ (x=2y)$
Odd(x) $\equiv \exists y\ (x=2y+1)$
Domain: Integers

Prove: "The square of every odd number is odd"

English proof of: $\forall x\,(\text{Odd}(x) \rightarrow \text{Odd}(x^2))$

Let x be an odd number.

Then x=2k+1 for some integer k (depending on x)

Therefore $x^2=(2k+1)^2= 4k^2+4k+1=2(2k^2+2k)+1$.

Since $2k^2+2k$ is an integer, x^2 is odd.

Proof by Contradiction: One way to prove $\neg p$

If we assume p and derive False (a contradiction), then we have proved $\neg p$.

1.	p	assumption
...		
3.	F	
4.	$p \rightarrow F$	direct Proof rule
5.	$\neg p \vee F$	equivalence from 4
6.	$\neg p$	equivalence from 5