CSE 311

Foundations of Computing I

Fall 2014

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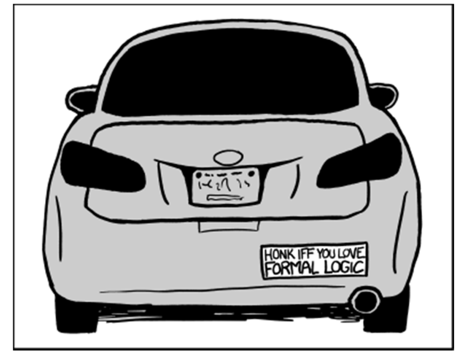
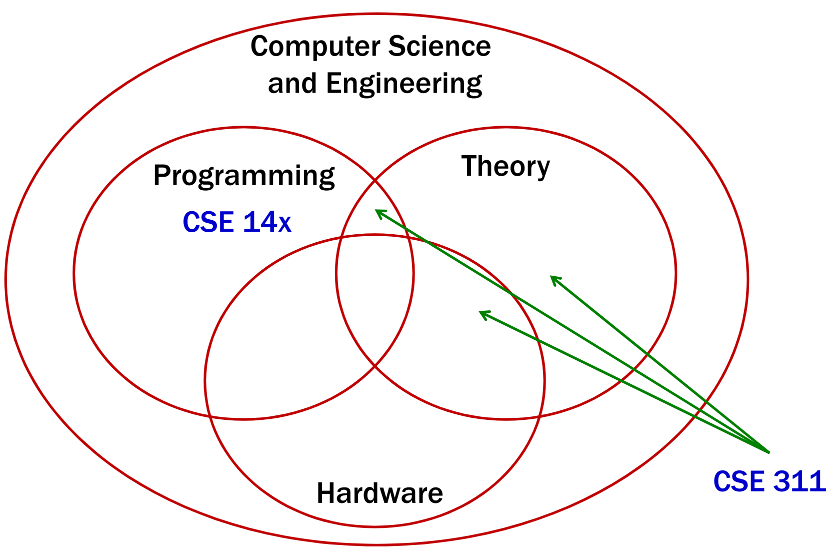
Lecture 1: Propositional Logic



Some Perspective



About the Course

We will study the *theory* needed for CSE:

Logic:
How can we describe ideas *precisely*?

Formal Proofs:
How can we be *positive* we're correct?

Number Theory:
How do we keep data *secure?*

Relations/Relational Algebra:
How do we store information?

Finite State Machines:
How do we design hardware and software?

Turing Machines:
Are there problems computers *can't* solve?

About the Course

It's about perspective!

- Example: Sudoku
  - Given *one*, solve it by hand
  - Given *most*, solve them with a program
  - Given *any*, solve it with computer science

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives…
- Fundamental structures for computer science

Administrivia

Instructors: Paul Beame and Adam Blank

Teaching assistants:

Antoine Bosselut	Nickolas Evans
Akash Gupta	Jeffrey Hon
Shawn Lee	Elaine Levey
Evan McCarty	Yueqi Sheng

Quiz Sections: Thursdays

(Optional) Book:
Rosen
Discrete Mathematics
6th or 7th edition
Can buy online for ~$50

Homework:
Due WED at start of class
Write up individually

Exams:
Midterm: Monday, November 3
Final: Monday, December 8
2:30-4:20 or 4:30-6:20
Non-standard time

Grading (roughly):
50% homework
35% final exam
15% midterm

All course information at http://www.cs.washington.edu/311

Logic: The Language of Reasoning

- Why not use English?
  - Turn right here…
  - Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo
  - We saw her duck
- "Language of Reasoning" like Java or English
  - Words, sentences, paragraphs, arguments…
  - Today is about **words** and **sentences**

Why Learn A New Language?

- Logic, as the "language of reasoning", will help us…
  - Be more **precise**
  - Be more **concise**
  - Figure out what a statement means more **quickly**

Propositions

- A **proposition** is a statement that
  - has a truth value, and
  - is "well-formed"

A proposition is a statement that has a truth value, and is "well-formed"...

- Consider these statements:
  - 2 + 2 = 5
  - The home page renders correctly in IE.
  - This is the song that never ends...
  - Turn in your homework on Wednesday.
  - This statement is false.
  - Akjsdf?
  - The Washington State flag is red.
  - Every positive even integer can be written as the sum of two primes.

Propositions

- A **proposition** is a statement that
  - has a truth value, and
  - is "well-formed"

- Propositional Variables: $p, q, r, s, \ldots$
- Truth Values: T for true, F for false

A Proposition

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

- What does this proposition mean?
- It seems to be built out of other, more basic propositions that are sitting inside it! What are they?

How are the basic propositions combined?

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

- RElephant : **"Roger is an orange elephant"**
- RTusks : **"Roger has tusks"**
- RToenails : **"Roger has toenails"**

Logical Connectives

- **Negation (not)** $\neg p$
- **Conjunction (and)** $p \wedge q$
- **Disjunction (or)** $p \vee q$
- **Exclusive or** $p \oplus q$
- **Implication** $p \rightarrow q$
- **Biconditional** $p \leftrightarrow q$

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"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant and (RToenails if RTusks) and (RToenails or RTusks or (RToenails and RTusks))

Some Truth Tables

p	$\neg p$

p	q	$p \wedge q$

p	q	$p \vee q$

p	q	$p \oplus q$

$p \rightarrow q$

- "If p, then q" is a **promise**:
  - Whenever p is true, then q is true
  - Ask "has the promise been broken"

p	q	p → q

If it's raining, then I have my umbrella

Suppose it's not raining…

$p \rightarrow q$

"I am a Pokémon master only if I have collected all 151 Pokémon"

Can we re-phrase this as if p, then q?

$p \rightarrow q$

Implication:

- *p* implies *q*
- whenever *p* is true *q* must be true
- if *p* then *q*
- *q* if *p*
- *p* is sufficient for *q*
- *p* only if *q*

p	q	p → q

Converse, Contrapositive, Inverse

- Implication: $p \rightarrow q$
- Converse: $q \rightarrow p$
- Contrapositive: $\neg q \rightarrow \neg p$
- Inverse: $\neg p \rightarrow \neg q$

How do these relate to each other?

Back to Roger's Sentence

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant ∧ (RToenails **if** RTusks) ∧ (RToenails ∨ RTusks ∨ (RToenails ∧ RTusks))

Define shorthand …

p : RElephant
q : RTusks
r : RToenails

Roger's Sentence with a Truth Table

p	q	r	$q \to r$	$p \wedge (q \to r)$	$r \vee q$	$r \wedge q$	$(r \vee q) \vee (r \wedge q)$	$p \wedge (q \to r) \wedge (r \vee q \vee (r \wedge q))$

More about Roger

Roger is only orange if whenever he either has tusks or toenails, he doesn't have tusks and he is an orange elephant."

- RElephant : **"Roger is an orange elephant"**
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More about Roger

Roger is only orange if whenever he either has tusks or toenails, he doesn't have tusks and he is an orange elephant."

(RElephant only if (whenever (RTusks **xor** RToenails) then not RTusks)) and RElephant

(RElephant → (**whenever** (RTusks ⊕ RToenails) then ¬ RTusks)) ∧ RElephant

p : RElephant
q : RTusks
r : RToenails

Roger's Second Sentence with a Truth Table

p	q	r	$q \oplus r$	$\neg q$	$((q \oplus r) \rightarrow \neg q)$	$p \rightarrow ((q \oplus r) \rightarrow \neg q)$	$(p \rightarrow ((q \oplus r) \rightarrow \neg q)) \wedge p$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Biconditional: $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*

p	q	$p \leftrightarrow q$