CSE 311

Foundations of Computing I

Fall 2014

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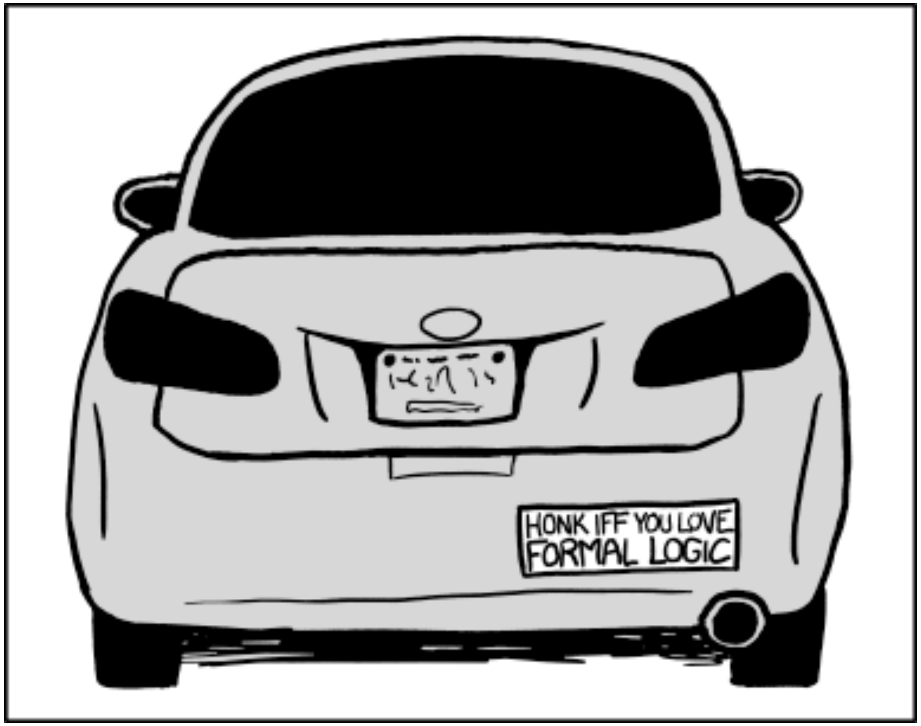
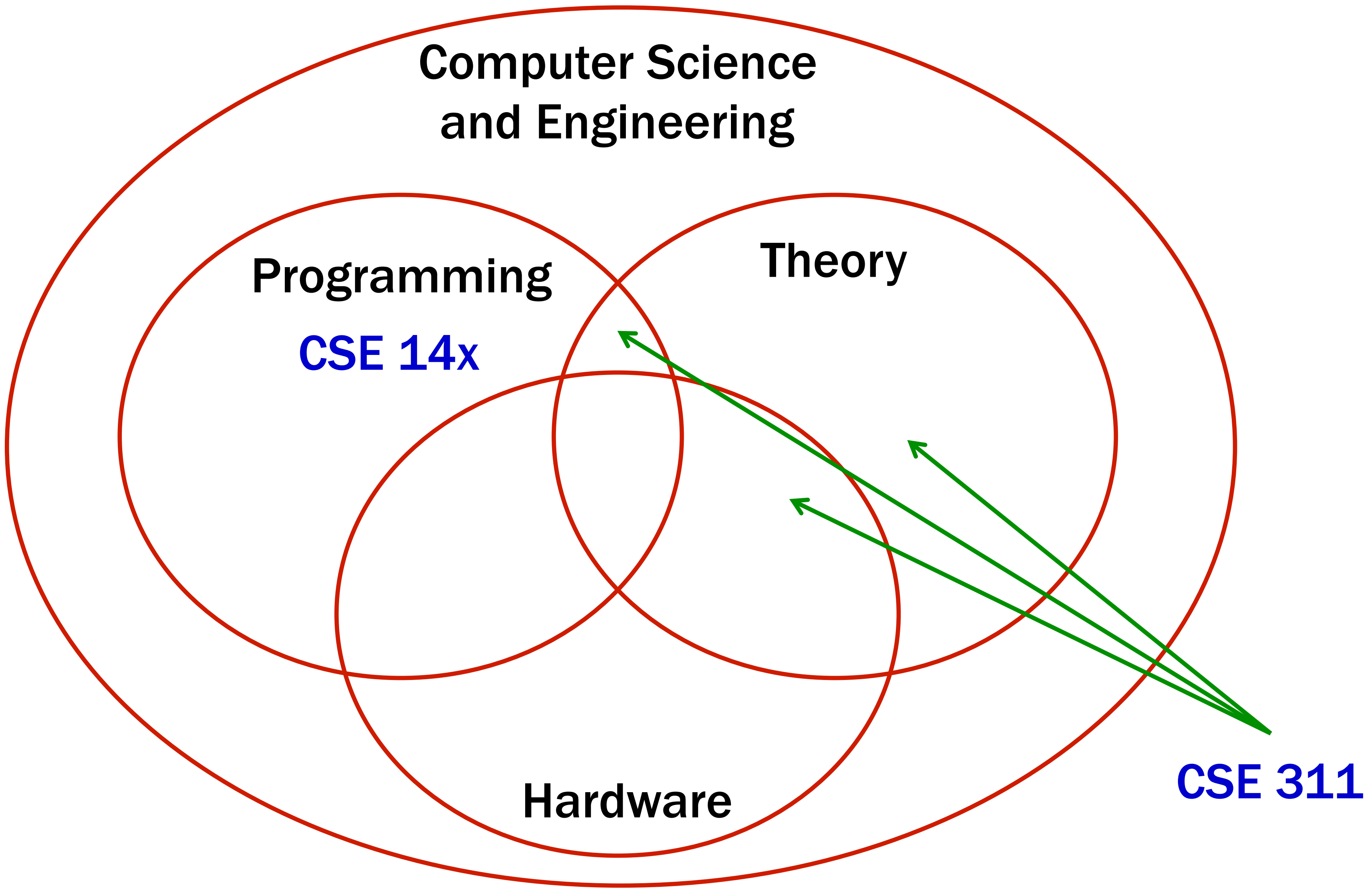
Lecture 1: Propositional Logic



Some Perspective



About the Course

We will study the *theory* needed for CSE:

Logic:

How can we describe ideas *precisely*?

Formal Proofs:

How can we be *positive* we're correct?

Number Theory:

How do we keep data *secure?*

Relations/Relational Algebra:

How do we store information?

Finite State Machines:

How do we design hardware and software?

Turing Machines:

Are there problems computers *can't* solve?

About the Course

It's about perspective!

- Example: Sudoku
  - Given *one*, solve it by hand
  - Given *most*, solve them with a program
  - Given *any*, solve it with computer science

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives...
- Fundamental structures for computer science

Administrivia

Instructors: Adam Blank and Paul Beame

Teaching assistants:

Antoine Bosselut	Nickolas Evans
Akash Gupta	Jeffrey Hon
Shawn Lee	Elaine Levey
Evan McCarty	Yueqi Sheng

Section: Thursdays

(Optional) Book:

- Rosen
- Discrete Mathematics
- 6th or 7th edition
- Can buy online for ~$50

Homework:

- Due WED at start of class
- Write up individually

Exams:

- Midterm: Monday, November 3
- Final: Monday, December 8
  - 2:30-4:20 or 4:30-6:20
  - Non-standard time

Grading (roughly):

- 50% homework
- 35% final exam
- 15% midterm

All course information at http://www.cs.washington.edu/311

Logic: The Language of Reasoning

- Why not use English?
  - Turn right here…
  - Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo
  - We saw her duck

Logic: The Language of Reasoning

- Why not use English?
  - Turn right here… (direction or time?)
  - Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo (words have multiple meanings)
  - We saw her duck (animal or crouch)
- "Language of Reasoning" like Java or English
  - Words, sentences, paragraphs, arguments…
  - Today is about **words** and **sentences**

Why Learn A New Language?

- Logic, as the “language of reasoning”, will help us…
  - Be more **precise**
  - Be more **concise**
  - Figure out what a statement means more **quickly**

Propositions

- A **proposition** is a statement that
  - has a truth value, and
  - is "well-formed"

A proposition is a statement that has a truth value, and is “well-formed”…

- Consider these statements:
  - 2 + 2 = 5
  - The home page renders correctly in IE.
  - This is the song that never ends…
  - Turn in your homework on Wednesday.
  - This statement is false.
  - Akjsdf?
  - The Washington State flag is red.
  - Every positive even integer can be written as the sum of two primes.

A proposition is a statement that has a truth value, and is "well-formed"…

- Consider these statements:
  - 2 + 2 = 5 (this is a statement! We can talk about false things!)
  - The home page renders correctly in IE. (this is a statement! We can talk about false things!)
  - This is the song that never ends… (this isn't well-formed; what does it mean? What is "…"?)
  - Turn in your homework on Wednesday. (this is a command, it doesn't have a truth value)
  - This statement is false. (this doesn't have a truth value)
  - Akjsdf? (this is not well-formed; it's gibberish)
  - The Washington State flag is red. (this is a statement!)
  - Every positive even integer can be written as the sum of two primes. (we may not know if this is true or not, but it's definitely a statement!)

Propositions

- A **proposition** is a statement that
  - has a truth value, and
  - is "well-formed"

- Propositional Variables: $p, q, r, s, \ldots$
- Truth Values: **T** for true, **F** for false

A Proposition

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

- **What does this proposition mean?**
- **It seems to be built out of other, more basic propositions that are sitting inside it! What are they?**

- RElephant : **"Roger is an orange elephant"**
- RTusks : **"Roger has tusks"**
- RToenails : **"Roger has toenails"**

How are the basic propositions combined?

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

- RElephant : **"Roger is an orange elephant"**
- RTusks : **"Roger has tusks"**
- RToenails : **"Roger has toenails"**

RElephant **and** (RToenails **if** RTusks) **and** (RToenails **or** RTusks **or** (RToenails **and** RTusks))

Logical Connectives

- Negation (not) $\neg p$
- Conjunction (and) $p \wedge q$
- Disjunction (or) $p \vee q$
- Exclusive or $p \oplus q$
- Implication $p \rightarrow q$
- Biconditional $p \leftrightarrow q$

Logical Connectives

- **Negation (not)** $\neg p$
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- **Implication** $p \rightarrow q$
- **Biconditional** $p \leftrightarrow q$

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant and (RToenails if RTusks) and (RToenails or RTusks or (RToenails and RTusks))

RElephant ∧ (RToenails if RTusks) ∧ (RToenails ∨ RTusks ∨ (RToenails ∧ Rtusks))

Some Truth Tables

p	$\neg p$

p	q	$p \wedge q$

p	q	$p \vee q$

p	q	$p \oplus q$

Some Truth Tables

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$p \to q$

- "If p, then q" is a **promise**:
  - Whenever p is true, then *q* is true
  - Ask "has the promise been broken"

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

If it's raining, then I have my umbrella

Suppose it's not raining…

First Question: It's not raining, and I don't bring my umbrella. Have I broken the promise?
Second Question: It's not raining, and I bring my umbrella. Have I broken the promise?
In both cases, the pre-requisite to my promise isn't met. So, I haven't in either case. In fact, the only case in which I've lied to you is when it's raining, but I don't have my umbrella.

$p \rightarrow q$

"I am a Pokémon master only if I have collected all 151 Pokémon"

Can we re-phrase this as if p, then q?

The sentence is saying collecting all the Pokémon is a pre-requisite to being a Pokémon master.

So…

"If I am a Pokémon master, I have all 151 Pokémon"

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication:

- p implies q
- whenever p is true q must be true
- if p then q
- q if p
- p is sufficient for q
- p only if q

Converse, Contrapositive, Inverse

- Implication: $p \rightarrow q$
- Converse: $q \rightarrow p$
- Contrapositive: $\neg q \rightarrow \neg p$
- Inverse: $\neg p \rightarrow \neg q$

How do these relate to each other?

Example:

p: x is divisible by 2

q: x is divisible by 4

$p \rightarrow q$	False
$q \rightarrow p$	True
$\neg q \rightarrow \neg p$	False
$\neg p \rightarrow \neg q$	True

Back to Roger's Sentence

"Roger is an orange elephant who has toenails if he has tusks, and has toenails, tusks, or both."

RElephant ∧ (RToenails if RTusks) ∧ (RToenails ∨ RTusks ∨ (RToenails ∧ RTusks))

Define shorthand ...

p : RElephant
q : RTusks
r : RToenails

$$(p \land (q \rightarrow r) \land (r \lor q \lor (r \land q))$$

Roger's Sentence with a Truth Table

p	q	r	$q \to r$	$p \wedge (q \to r)$	$r \vee q$	$r \wedge q$	$(r \vee q) \vee (r \wedge q)$	$p \wedge (q \to r) \wedge (r \vee q \vee (r \wedge q))$

Roger's Sentence with a Truth Table

p	q	r	$q \to r$	$p \wedge (q \to r)$	$r \vee q$	$r \wedge q$	$(r \vee q) \vee (r \wedge q)$	$p \wedge (q \to r) \wedge (r \vee q \vee (r \wedge q))$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T	F
T	F	T	T	T	T	F	T	T
T	F	F	T	T	F	F	F	F
F	T	T	T	F	T	T	T	F
F	T	F	F	F	T	F	T	F
F	F	T	T	F	T	F	T	F
F	F	F	T	F	F	F	F	F

More about Roger

Roger is only orange if whenever he either has tusks or toenails, he doesn't have tusks and he is an orange elephant.”

- RElephant : **“Roger is an orange elephant”**
- RTusks : **“Roger has tusks”**
- RToenails : **“Roger has toenails”**

(RElephant **only if** (**whenever** (RTusks **xor** RToenails) **then not** RTusks)) **and** RElephant

More about Roger

Roger is only orange if whenever he either has tusks or toenails, he doesn't have tusks and he is an orange elephant."

↓

(RElephant **only if** (**whenever** (Rtusks **xor** RToenails) **then not** RTusks)) **and** RElephant

↓

(RElephant → (whenever (RTusks ⊕ RToenails) then ¬ RTusks)) ∧ RElephant

p : RElephant
q : RTusks
r : RToenails

↓

$$(p \to ((q \oplus r) \to \neg q)) \land p$$

Roger's Second Sentence with a Truth Table

p	q	r	$q \oplus r$	$\neg q$	$((q \oplus r) \to \neg q)$	$p \to ((q \oplus r) \to \neg q)$	$(p \to ((q \oplus r) \to \neg q)) \wedge p$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Roger's Second Sentence with a Truth Table

p	q	r	$q \oplus r$	$\neg q$	$((q \oplus r) \to \neg q)$	$p \to ((q \oplus r) \to \neg q)$	$(p \to ((q \oplus r) \to \neg q)) \wedge p$
T	T	T	F	F	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	F	T	T	F
F	T	F	T	F	F	T	F
F	F	T	T	T	T	T	F
F	F	F	F	T	T	T	F

Biconditional: $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*

p	q	$p \leftrightarrow q$

Biconditional: $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Next Time...

Bill

V

Twins