CSE 311: Foundations of Computing I

Section: Relations, CFGs, and DFAs Solutions

CFGs

Construct CFGs for the following languages:

(a) All binary strings that end in 00.

Solution:

$$\mathsf{S} \rightarrow 0\mathsf{S} \mid 1\mathsf{S} \mid 00$$

(b) All binary strings that contain at least three 1's.

Solution:

$$\begin{aligned} \mathsf{S} &\rightarrow 0\mathsf{S} \mid 1\mathsf{T} \\ \mathsf{T}_1 &\rightarrow 0\mathsf{T}_1 \mid 1\mathsf{T}_2 \\ \mathsf{T}_2 &\rightarrow 0\mathsf{T}_2 \mid 1\mathsf{T}_3 \\ \mathsf{T}_3 &\rightarrow 0\mathsf{T}_3 \mid 1\mathsf{T}_3 \mid \varepsilon \end{aligned}$$

(c) All binary strings with an equal number of 1's and 0's.

Solution:

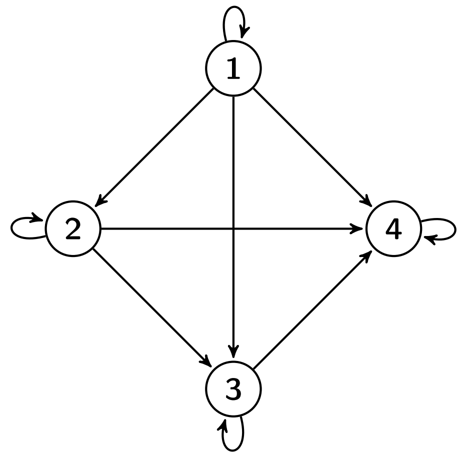
$$\mathsf{S} \rightarrow 0\mathsf{S}1\mathsf{S} \mid 1\mathsf{S}0\mathsf{S} \mid \varepsilon$$

$$\mathsf{S} \rightarrow \mathsf{S}\mathsf{S} \mid 0\mathsf{S}1 \mid 1\mathsf{S}0 \mid \varepsilon$$

Relations

(a) Draw the transitive-reflexive closure of $\{(1, 2), (2, 3), (3, 4)\}$.

Solution:



(b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.

Solution: Suppose $(a, b) \in R$. Since R is reflexive, we know $(b, b) \in R$ as well. Since there is a b such that $(a, b) \in R$ and $(b, b) \in R$, it follows that $(a, b) \in R^2$. Thus, $R \subseteq R^2$.

(c) Consider the relation $R = \{(x, y) \ : \ x = y + 1\}$ on $\mathbb{N}$. Is R reflexive? Transitive? Symmetric? Anti-symmetric?

Solution: It isn't reflexive, because $1 \neq 1 + 1$; so, $(1, 1) \notin R$. It isn't symmetric, because $(2, 1) \in R$ (because $2 = 1 + 1$), but $(1, 2) \notin R$, because $1 \neq 2 + 1$. It isn't transitive, because note that $(3, 2) \in R$ and $(2, 1) \in R$, but $(3, 1) \notin R$. It is anti-symmetric, because consider $(x, y) \in R$ such that $x \neq y$. Then, $x = y + 1$ by definition of R. However, $(y, x) \notin R$, because $y = x - 1 \neq x + 1$.

(d) Consider the relation $S = \{(x, y) \mid x^2 = y^2\}$ on $\mathbb{R}$. Prove that S is reflexive, transitive, and symmetric.

Solution: Consider $x \in \mathbb{R}$. Note that by definition of equality, $x^2 = x^2$; so, $(x, x) \in R$; so, R is reflexive.

Consider $(x, y) \in R$. Then, $x^2 = y^2$. It follows that $y^2 = x^2$; so, $(y, x) \in R$. So, R is symmetric.
Suppose $(x, y) \in R$ and $(y, z) \in R$. Then, $x^2 = y^2$, and $y^2 = z^2$. Since equality is transitive, $x^2 = z^2$. So, $(x, z) \in R$. So, R is transitive.

DFAs

Construct a DFA for the language of all binary strings, where $\Sigma = \{0, 1, 2\}$.

Solution: Omitted.