CSE 311: Foundations of Computing I

Section: Structural Induction and Regular Expressions

Structural Induction

(a) Recall the following definitions:

$$\begin{aligned} \mathsf{len}(\varepsilon) &= 0 \\ \mathsf{len}(wa) &= \mathsf{len}(w) + 1, \text{for } w \in \Sigma^*, a \in \Sigma \end{aligned}$$

$$\begin{aligned} x \bullet \varepsilon &= x, \text{for } x \in \Sigma^* \\ x \bullet wa &= (x \bullet w)a, \text{for } x \in \Sigma^*, a \in \Sigma \end{aligned}$$

Consider the following recursive definition:

$$\begin{aligned} \texttt{stutter}(\varepsilon) &= \varepsilon \\ \texttt{stutter}(wa) &= \texttt{stutter}(w) \bullet aa, \text{for } w \in \Sigma^*, a \in \Sigma \end{aligned}$$

Prove that $\mathsf{len}(\texttt{stutter}(w)) = 2\mathsf{len}(w)$ for all $w \in \Sigma^*$.

Regular Expressions

(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3.

(c) Write a regular expression that matches all binary strings that contain the substring "111", but not the substring "000".