

CSE 311: Foundations of Computing I

Section: Sets and Modular Arithmetic

How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say so.

- (a) $A = \{1, 2, 3, 2\}$
- (b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \dots\}$
- (c) $C = A \times (B \cup \{7\})$
- (d) $D = \emptyset$
- (e) $E = \{\emptyset\}$
- (f) $F = \mathcal{P}(\{\emptyset\})$

Set = Set

Prove the following set identities.

- (a) Let the universal set be \mathcal{U} . Prove $\overline{\overline{X}} = X$ for any set X .
- (b) Prove $(A \oplus B) \oplus B = A$ for any sets A, B .
- (c) Prove $A \cup B \subseteq A \cup B \cup C$ for any sets A, B, C .
- (d) Let the universal set be \mathcal{U} . Prove $A \cap \overline{B} \subseteq A \setminus B$ for any sets A, B .

Casting Out Nines

Let $n \in \mathbb{N}$. Prove that if $n \equiv 0 \pmod{9}$, then the sum of the digits of n is a multiple of 9.

Modular Arithmetic

- (a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.
- (b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.