CSE 311: Foundations of Computing I

Section: FOL and Inference

Translate to Logic

Express each of these system specifications using predicate, quantifiers, and logical connectives.

(a) Every user has access to an electronic mailbox.

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

(d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

Translate to English

Translate these system specifications into English where $F(p)$ is "Printer p is out of service", $B(p)$ is "Printer p is busy", $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued". Let the domain be all printers.

(a) $\exists p\ (F(p) \land B(p)) \to \exists j\ L(j)$

(b) $\forall p\ B(p) \to \exists j\ Q(j)$

(c) $\exists j\ (Q(j) \land L(j)) \to \exists p\ F(p)$

(d) $(\forall p\ B(p) \land \forall j\ Q(j)) \to \exists j\ L(j)$

Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other (or if they are equivalent).

(a) $\forall x\ \forall y\ P(x, y)$ $\qquad$ $\forall y\ \forall x\ P(x, y)$

(b) $\exists x\ \exists y\ P(x, y)$ $\qquad$ $\exists y\ \exists x\ P(x, y)$

(c) $\forall x\ \exists y\ P(x, y)$ $\qquad$ $\forall y\ \exists x\ P(x, y)$

(d) $\forall x\ \exists y\ P(x, y)$ $\qquad$ $\exists x\ \forall y\ P(x, y)$

Formal Proofs

For each of the following part, write *formal proofs*.

(a) Prove $\forall x\ (R(x) \land S(x))$ given $\forall x\ (P(x) \to (Q(x) \land S(x)))$, and $\forall x\ (P(x) \land R(x)$.

(b) Prove $\exists x\ \neg R(x)$ given $\forall x\ (P(x) \lor Q(x))$, $\forall x\ (\neg Q(x) \lor S(x))$, $\forall x\ (R(x) \to \neg S(x))$, and $\exists x\ \neg P(x)$.

English Proof

Prove that if a real number $x \neq 0$, then $x^2 + \frac{1}{x^2} \geq 2$.

Primality Checking

When brute forcing if the number p is prime, you only need to check possible factors up to $\sqrt{p}$. In this problem, you'll prove why that is the case. Prove that if $n = ab$, then either a or b is at most $\sqrt{n}$.
(*Hint:* You want to prove an implication; so, start by assuming $n = ab$. Then, continue by writing out your assumption for contradiction.)