CSE 311: Foundations of Computing I

QuickCheck: Sets Solutions (due Thursday, October 16)

0. Sets All Folks!

Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Note that while a Venn Diagram is useful, it is not a proof.

Solution:

$$
\begin{aligned}
A \cap (B \cup C) &= \{x \; : \; x \in A \cap (B \cup C)\} && \text{[Set Comprehension]} \\
&= \{x \; : \; x \in A \wedge x \in B \cup C\} && \text{[Definition of } \cap \text{]} \\
&= \{x \; : \; x \in A \wedge (x \in B \vee x \in C)\} && \text{[Definition of } \cup \text{]} \\
&= \{x \; : \; (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)\} && \text{[Distributivity]} \\
&= \{x \; : \; (x \in A \cap B) \vee (x \in A \cap C)\} && \text{[Definition of } \cap \text{]} \\
&= \{x \; : \; x \in (A \cap B) \cup (A \cap C)\} && \text{[Definition of } \cup \text{]} \\
&= (A \cap B) \cup (A \cap C) && \text{[Set Comprehension]}
\end{aligned}
$$