CSE 311: Foundations of Computing I

QuickCheck: FOL and Inference Solutions (due Thursday, October 9)

0. Oddly Even

Let Even(x) be $\exists y\ x = 2y$, and let Odd(x) be $\exists y\ x = 2y + 1$.

(a) Translate the statement

$$\forall x\ \forall y\ ((\text{Odd}(x) \land \text{Odd}(y)) \rightarrow \text{Even}(x + y))$$

into English.

Solution: For all integers x, y, if x and y are odd, then $x + y$ is even.

(b) Prove the statement from part (a) using a *formal proof*.

1. Let x be an integer.
2. Let y be an integer.

Solution:

1.	Let x be an integer.	
2.	Let y be an integer.	
3.	Odd(x) $\land$ Odd(y)	[Assumption]
4.	Odd(x)	[Elim $\land$: 3]
5.	$\exists k\ x = 2k + 1$	[Definition of Odd, 4]
6.	$x = 2k + 1$	[Elim $\exists$: 5]
7.	Odd(y)	[Elim $\land$: 3]
8.	$\exists k\ y = 2k + 1$	[Definition of Odd, 7]
9.	$y = 2j + 1$	[Elim $\exists$: 8]
10.	$x + y = 2k + 1 + 2j + 1$	[Algebra: 6, 9]
11.	$x + y = 2(k + j + 1)$	[Algebra: 10]
12.	$\exists r\ x + y = 2r$	[Intro $\exists$: 11]
13.	Even($x + y$)	[Definition of Even, 12]
14.	Odd(x) $\land$ Odd(y) $\rightarrow$ Even($x + y$)	[Direct Proof Rule]