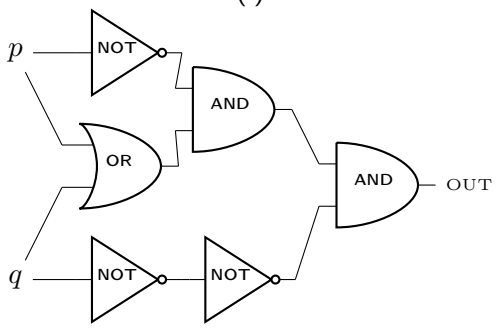
CSE 311: Foundations of Computing I

QuickCheck: Gates and Equivalence Solutions (due Thursday, October 2)

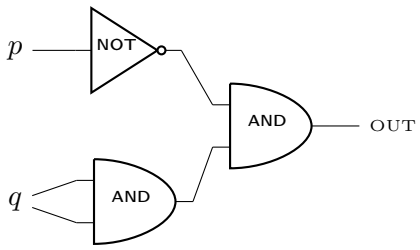
0. If you turn the paper horizontally, the circuits look like robots (:

(a) Convert each of the following circuits to logical expressions.

(i)



(ii)



Solution:

(i) $((\neg p) \wedge (p \vee q)) \wedge \neg\neg q$

(ii) $\neg p \wedge (q \wedge q)$

(b) Prove that (i) and (ii) are equivalent using a truth table.

Solution:

p	q	$(\neg p \wedge (p \vee q))$	$(\neg p \wedge (p \vee q)) \wedge \neg\neg q$	$\neg p \wedge (q \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	T
F	F	F	F	F

(c) Prove that (i) and (ii) are equivalent using propositional equivalences. See the back page for a full list of them.

Solution:

$$
\begin{aligned}
(\neg p \wedge (p \vee q)) \wedge \neg\neg q &\equiv (\neg p \wedge (p \vee q)) \wedge q && \text{[Proven in lecture]} \\
&\equiv \neg p \wedge ((p \vee q) \wedge q) && \text{[Associative]} \\
&\equiv \neg p \wedge (q \wedge (q \vee p)) && \text{[Commutative twice]} \\
&\equiv \neg p \wedge q && \text{[Absorbtion]} \\
&\equiv \neg p \wedge (q \wedge q) && \text{[Idempotent]}
\end{aligned}
$$

(d) Write (i) and (ii) as expressions in boolean algebra.

Solution:

(i) (p' • (p + q)) • q''

(ii) p' • (q • q)