CSE 311: Foundations of Computing I
Homework 9 (due Wednesday, December 5)

0. String Search (20 points)
Use the method given in class to design a linear-time algorithm to determine all occurrences of the string 10100101001 in strings over the alphabet $\{0, 1\}$. Submit your answer using the FSM website; (This isn't quite up yet; enjoy your Thanksgiving!)

1. Diagonalization (30 points)
Let B be the set of all infinite binary sequences. Show that B is uncountable using a proof by diagonalization.

2. Countabilitity (20 points)
An *integer quadratic polynomial* is a function of the form $f(x) = ax^2 + bx + c$ where $a \neq 0, b, c \in \mathbb{Z}$. Show that the set of all integer quadratic polynomials is countable.

3. Undecidability (30 points)
Consider the set **Identity**:

Identity $= \{\texttt{CODE}(\mathbf{P}) : \mathbf{P}$ is a Java program with domain and codomain $\mathbb{N}$, and for all inputs x, $\mathbf{P}(x) = x\}$

For example, the following programs all *are not* in **Identity**:

```
1    public nat f(nat x) {
2        System.out.println("hello");
3        return x²;
4    }
```

```
1    public void f(nat x) {
2        System.out.println("I have no output");
3    }
```

```
1    public nat f(nat x) {
2        if (x % 2 == 0) {
3            return 2x;
4        }
5        else {
6            return x;
7        }
8    }
```

While the following programs all *are* in **Identity**:

```
1    public nat f(nat x) {
2        return x;
3    }
```

```
1    public nat f(nat x) {
2        System.out.println("3");
3        return x;
4    }
```

```
1    public nat f(nat x) {
2        if (true) {
3            return x;
4        }
5        else {
6            return 5;
7        }
8    }
```

Show that **Identity** is undecidable using the fact that the Halting Problem is undecidable.