CSE 311: Foundations of Computing I

Homework 5 (due Wednesday, October 29)

Directions: *Write up carefully argued solutions to the following problems. The first task is to be complete and correct. The more subtle task is to keep it simple and succinct. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use any results proven in lecture without proof. Anything else must be argued rigorously. Unless otherwise specified, all answers are expected to be given in closed form.*

0. Sum r Close, and Sum r Far (20 points)

Please complete this question on a *separate piece of paper*. The reason we're asking you to do this is so that you can have at least one graded induction question back before the midterm. Putting this answer on a separate piece of paper will allow us to return this question before the rest of the homework has been graded.

Prove the following inequality for all integers $n \geq 2$:

$$\sum_{i=2}^{n} \frac{1}{(i-1)i} < 1$$

Hint: What is the sum exactly?

1. GCDon't (10 points)

(a) [1 Point] Compute $\gcd(0, 12^{32})$.

(b) [3 Points] Compute $\gcd(135, 67)$ using Euclid's Algorithm.

(c) [6 Points] Compute $\gcd(91, 434)$ using Euclid's Algorithm. Show your intermediate results.

2. Sesrevni (20 points)

(a) [5 Points] Compute the multiplicative inverse of 121 modulo 17 using the Extended Euclidean Algorithm. Show your intermediate results.

(b) [5 Points] Find all solutions x with $0 \leq x < 43$ to the following equation:

$$67x \equiv 3 \pmod{43}$$

Show your intermediate results.

(c) [5 Points] Prove that there are no solutions to the following equation:

$$9x \equiv 2 \pmod{15}$$

(d) [5 Points] On the last homework you showed that $ca \equiv cb \pmod{cm}$ implies $a \equiv b \pmod{m}$. The other direction is true as well. Using this fact, find:

$$10x \equiv 70 \pmod{135}$$

3. Modular$^{\text{Exponentiation}^{\text{Question}}}$ (5 points)

Compute $3^{97} \bmod 100$ using the modular exponentation algorithm. Show your intermediate results. How many multiplications does the algorithm use for this computation?

4. Why Induction Matters (5 points)

Claim: For all $n \geq 1$, $\gcd(n^5 - 5, (n+1)^5 - 5) = 1$.
Disprove the claim.
Hint: Use a computer.

5. Abel's Inequality (20 points)

Prove Abel's Inequality for all $n \in \mathbb{N}$ and $x \in \mathbb{R}$, $x > -1$: $(1+x)^n \geq 1 + nx$

6. Your Average Induction (20 points)

Consider the following code:

```
public double average(ArrayList<Integer> grades, int numberOfGrades) {
   if (numberOfGrades == 1) {
      return grades.get(0);
   }
   else {
      return (
         (average(grades, numberOfGrades − 1) ∗ (numberOfGrades − 1)) +
         grades.get(numberOfGrades − 1)
      ) / numberOfGrades;
   }
}
```

Let A be an arbitrary, non-empty `ArrayList<Integer>`, and let $n = \texttt{A.size()}$.

Prove that $\texttt{average}(A, n) = \displaystyle\sum_{i=0}^{n-1} \frac{A_i}{n}$, where $A_i = A.\texttt{get(i)}$ for $0 \leq i < n$ for all $n \geq 1$.

7. EXTRA CREDIT: Sum Products (-NoValue- points)

Find a simple closed form (no summations or products) for the following equation and prove your closed form correct for all positive integers n:

$$\sum_{\substack{A \\ \varnothing \subset A \subseteq [n]}} \prod_{a \in A} \frac{1}{a}$$