CSE 311: Foundations of Computing I

Homework 4 (due Wednesday, October 22)

Directions: *Write up carefully argued solutions to the following problems. The first task is to be complete and correct. The more subtle task is to keep it simple and succinct. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use any results proven in lecture without proof. Anything else must be argued rigorously. Unless otherwise specified, all answers are expected to be given in closed form.*

0. $X - Y + Y = X$ (22 points)

Let $X, Y \subseteq \mathbb{Z}$, and $x, y \in \mathbb{Z}$. Consider the following possible identity:

$$(X \setminus \{x\}) \cup \{x\} \stackrel{?}{=} X$$

(a) [10 Points] Prove or disprove $(X \setminus \{x\}) \cup \{x\} \subseteq X$.

(b) [10 Points] Prove or disprove $X \subseteq (X \setminus \{x\}) \cup \{x\}$.

(c) [2 Points] Does $(X \setminus \{x\}) \cup \{x\} = X$?

1. Powerful Sets (22 points)

Let $X, Y \subseteq \mathbb{Z}$, and $x, y \in \mathbb{Z}$. Consider the following possible identity:

$$\mathcal{P}(X \cup Y) \stackrel{?}{=} \mathcal{P}(X) \cup \mathcal{P}(Y) \cup \mathcal{P}(X \cap Y)$$

(a) [10 Points] Prove or disprove $\mathcal{P}(X \cup Y) \subseteq \mathcal{P}(X) \cup \mathcal{P}(Y) \cup \mathcal{P}(X \cap Y)$.

(b) [10 Points] Prove or disprove $\mathcal{P}(X) \cup \mathcal{P}(Y) \cup \mathcal{P}(X \cap Y) \subseteq \mathcal{P}(X \cup Y)$

(c) [2 Points] Does $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y) \cup \mathcal{P}(X \cap Y)$?

2. Minus Minus Minus (5 points)

Let the domain of discourse be all sets.

Disprove $\forall A\, \forall B\, \forall C\ (A \setminus B) \setminus C = A \setminus (B \setminus C)$ for sets A, B, and C

3. Cartesian Elimination (15 points)

Let A, B, and C be non-empty sets. Prove that $(A \times B = A \times C) \rightarrow B = C$.
What happens if A is empty?

4. Now you c me, now you don't (10 points)

Let a and b be integers, and let c and m be positive integers. Prove that if $ac \equiv bc \pmod{cm}$, then $a \equiv b \pmod{m}$.

5. Prime Examples (11 points)

Prove that for any prime $p > 3$, either $p \equiv 1 \pmod 6$ or $p \equiv 5 \pmod 6$.

6. 11 Modom (15 points)

We say an integer is *palindromic* if the digits read the same when written forward or backward. Prove that every palindromic integer with an even number of digits is divisible by 11.

Hint 1: $10 \equiv -1 \pmod{11}$
Hint 2: Use the base-10 representation of the number as a summation.

7. EXTRA CREDIT: Beetle Juice (-NoValue- points)

Each square of a 9×9 chessboard has a beetle on it. Every time a horn is blown, every beetle crawls diagonally onto a neighboring square. As a result, it is possible for many beetles to be on the same square of the chessboard. Find the minimal possible number of free squares after a single blow of the horn, and prove that your answer is a correct lower bound (i.e. that there cannot be fewer free squares).
(*Hint:* Consider marking parts of the chessboard with different colors.)
(*Challenge:* What about after more than just one move?)