CSE 311: Foundations of Computing I

Homework 3 (due Wednesday, October 15)

Directions: *Write up carefully argued solutions to the following problems. The first task is to be complete and correct. The more subtle task is to keep it simple and succinct. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use any results proven in lecture without proof. Anything else must be argued rigorously. Unless otherwise specified, all answers are expected to be given in closed form.*

0. Infer *That*! (10 points)

(a) [8 Points] Using the logical inference rules and equivalences we have given, write a *formal* proof that you can infer $r \rightarrow \neg q$ from $(p \vee r) \rightarrow (q \rightarrow s)$, t, and $(r \wedge t) \rightarrow \neg s$.

(b) [2 Points] How many rows would you need if you tried to do this using a truth table?

1. Mind Your P's and Q's (10 points)

Using the logical inference rules and equivalences we have given, write a *formal* proof that given $\forall x\ (Q(x) \rightarrow \neg P(x))$, $\forall x\ ((P(x) \wedge \neg Q(x)) \rightarrow R(x))$, and $\exists x\ P(x)$, you can conclude that $\exists x\ R(x)$.

2. Algebra-Shmalgebra (13 points)

Consider the following theorem and "spoof":

Theorem: Find all the solutions to the equation

$$(x-2)^2(y-4) + 32(x-2)(y-4) = 5(x-2)y$$

in the domain of all real numbers.

"Spoof:" First, we note that the original equation is the same as $(y-4)((x-2)^2+32(x-2)) = 5(x-2)y$. Then, factoring out and dividing by $y-4$, we get $(x-2)^2 + 32(x-2) = \frac{5(x-2)y}{y-4}$. Factoring out $x-2$ gives us $\frac{(x-2)((x-2)+32)}{x-2} = \frac{5y}{y-4}$. Then, dividing by $x-2$ results in $(x-2)+32 = \frac{5y}{y-4}$. Finally, we derive that all real number solutions (x, y) of the equation satisfy $x = \frac{5y}{y-4} - 30$.

(a) [3 Points] Is the conclusion of the "spoof" correct? Explain why or why not.

(b) [10 Points] The "spoof" makes a critical error in at least one step. Explain which step(s) are faulty and why.

3. Spoofclusions (13 points)

Theorem: Given $(p \wedge r) \rightarrow (q \wedge r)$, $\neg s \rightarrow (r \wedge q)$, and $s \rightarrow (p \wedge q)$, prove q.

"Spoof:"

1.	$(p \wedge r) \rightarrow (q \wedge r)$	[Given]
2.	$\neg s \rightarrow (r \wedge q)$	[Given]
3.	$s \rightarrow (p \wedge q)$	[Given]
4.	$\neg s \rightarrow q$	[Elim $\wedge$: 2]
5.	$p \rightarrow (q \wedge r)$	[Elim $\wedge$: 1]
6.	s [Assumption]	
7.	$p \wedge q$ [MP: 6, 3]	
8.	p [Elim $\wedge$: 7]	
9.	$q \wedge r$ [MP: 8, 5]	
10.	q [Elim $\wedge$: 9]	
11.	$s \rightarrow q$	[Direct Proof Rule]
12.	$(s \rightarrow q) \wedge (\neg s \rightarrow q)$	[Intro $\wedge$: 4, 11]
13.	$(\neg s \vee q) \wedge (\neg\neg s \vee q)$	[Law of Implication]
14.	$(q \vee \neg s) \wedge (q \vee \neg\neg s)$	[Commutativity]
15.	$q \wedge (\neg s \vee \neg\neg s)$	[Distributivity]
16.	$q \wedge T$	[Law of Negation]
17.	q	[Domination]

(a) [3 Points] Is the conclusion of the "spoof" correct? Explain why or why not.

(b) [10 Points] The "spoof" makes a critical error in at least one step. Explain which step(s) are faulty and why.

4. Oddly Prime (10 points)

Determine whether the following statement is *true* or *false*. Then, provide an *English proof* of your answer.

Write down the odd numbers starting with 43. Circle 43, delete one number, circle 47, delete two numbers, circle 53, delete three numbers, circle 61, and so on:

43	45	47	49	51	53	55	57	59	61	63	65	67	69	71
73	75	77	79	81	83	85	87	89	91	93	95	97	99	101
103	105	107	109	111	113	115	117	119	121	123	125	127	129	

Claim: The circled numbers in this sequence are all prime.

5. $\forall$ easy problems, $\exists$ a reason (10 points)

Prove or disprove each of the following statements using *English proofs*.

(a) [5 Points] Let SidesOf(x, y) be "the number of sides of x is y", Number(x) be "x is a number, and Shape(y) be "y is a shape." For this part, let the domain consist of the numbers 0 and 1, and the shapes triangle and square.

$$\forall x\ (\text{Number}(x) \rightarrow (\exists y\ (\text{Shape}(y) \rightarrow (\text{SidesOf}(y, x+4) \leftrightarrow (\text{Shape}(x) \vee \text{Number}(x))))))$$

(b) [5 Points] For this part, let the domain of discourse be non-negative integers.

$$\exists x\ \forall y\ ((y > 0) \rightarrow (2x < y))$$

6. The Odds It's Even (10 points)

For this question, let the domain of discourse be non-negative integers.

Recall:

- Even$(n) \equiv \exists k\ n = 2k$
- Odd$(n) \equiv \exists k\ n = 2k + 1$

Prove using an *english proof* that if n, m are odd, then nm is odd.

7. Oddly Even, again (19 points)

Let m and n be two integers. Show that $m^3 - n^3$ is even if and only if $m - n$ is even.

(a) [9 Points] Prove using a *formal proof* that $m - n$ is even $\rightarrow m^3 - n^3$ is even.

(b) [10 Points] Prove using an *English proof* that $m^3 - n^3$ is even $\rightarrow m - n$ is even.

8. Feed Us Feedback; we're hungry! (5 points)

Please go to `http://tinyurl.com/cse311-14au-feedback1` and fill out the *anonymous* feedback form. On the last page (once you have submitted), you will see a word marked as the "keyword"; write this keyword down on your homework to tell us you actually filled out the feedback, and you will get 5/5 for this question.

9. EXTRA CREDIT: Arrr! (-NoValue- points)

Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins.

On their ship, they decide to split the coins using the following scheme:

- The first pirate in alphabetical order becomes the chief pirate.
- The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way.
- Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain.

Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent, what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.