CSE 311: Foundations of Computing I

Homework 2 (due Wednesday, October 8)

Directions: *Write up carefully argued solutions to the following problems. The first task is to be complete and correct. The more subtle task is to keep it simple and succinct. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use any results proven in lecture without proof. Anything else must be argued rigorously. Unless otherwise specified, all answers are expected to be given in closed form.*

> **Note (10/03/14 5:00 PM):** There were some poorly defined domains; sorry about that. They should be fixed now.

0. Things Are Like. . . Other Things (8 points)

Prove that $(p \to q) \vee (r \wedge q) \equiv \neg(r \wedge (\neg q \wedge p)) \wedge \neg((p \wedge \neg r) \wedge \neg q)$ using equivalances.

1. Mammalian CS Majors (11 points)

For this question, let the domain of discourse consist of all mammals.

(a) [8 Points] Does there exist a person such that if that person is a CSE major, then everybody is a CSE major? Translate into logical notation, and then explain why it is true or false. Explain your answer.

Note: Your solution should work regardless of who the people in the universe are, as long as there is at least one.

(b) [3 Points] What happens if there are no people? Is the statement true or false? Explain why or why not.

2. Counting Courses with Combinational Logic (12 points)

In lecture 3, we considered a combinational logic example about days of class. We didn't figure out what `c1` was. Find a boolean expression for `c1`, and simplify it using boolean algebra axioms and theorems. Make sure to cite which axioms and theorems you are using when simplifying. Then, re-write the expression for `c1` in DNF and CNF.

3. Bananas, 311, and Bunnies (20 points)

It is often useful to assert that something exists and is unique. For instance,

There exists a unique course at UW with the course number CSE 311.

(a) [4 Points] Let the domain of discourse be all bananas. Let B be a constant representing "the purple banana in Adam's office." Translate "If there is a banana that is purple, then it is the purple banana in Adam's office." into first-order logic. Don't forget to define predicates.

(b) [4 Points] Let the domain of discourse be all university courses. Now, use an idea similar to part (a) to translate the sentence about the CSE 311 course number into first-order logic. Don't forget to define predicates.

For the remaining parts, we define the following predicates:

- Let $R(x)$ be "x is a rabbit."
- Let $H(x)$ be "x hops."

For these parts, let the domain be mammals. Translate each of the following into English.

(c) [3 Points] $\exists x(R(x) \land H(x))$

(d) [3 Points] $\forall x(R(x) \rightarrow H(x))$

(e) [3 Points] $\forall x(R(x) \land H(x))$

(f) [3 Points] $\exists x(R(x) \rightarrow H(x))$

4. $X \cdot Y$ (15 points)

In this question, you will construct a circuit that takes a pair of two-bit integers $(x_1x_0)_2$ and $(y_1y_0)_2$ and computes the four output bits for their integer product.

(a) [5 Points] Give sum-of-products forms for the two output bits of the product, $(a_1a_0)_2$, of $(x_1x_0)_2$ and $(y_0)_2$. Do the same for $(x_1x_0)_2$ and $(y_1)_2$ yielding $(b_1b_0)_2$. These are the bits produced as part of applying the usual elementary school method for multiplying numbers.

(b) [5 Points] Use the minimized sum-of-products forms for one-bit adders given in class, together with the results of the above two products to produce sum-of-products forms for the output bits z_3, z_2, z_1, z_0. Some of the inputs you give to the one-bit adders may be constants. Use Boolean algebra to minimize the resulting sum-of-products form as a sum-of-products using only x_1, x_0, y_1, y_0.

(c) [5 Points] Draw circuit diagrams for the results.

5. The Parentheses Matter! (10 points)

Give examples of predicates and domains such that the statements

$$\forall x\ (P(x) \lor Q(x))$$

and

$$\forall x\ P(x) \lor \forall x\ Q(x)$$

are not equivalent. Also give an example of predicates and domains where they *are* equivalent.

6. Axioms, Addition, and Algebra (8 points)

Prove that $X' + ((X + Y) \bullet X)' = X'$ using the axioms and theorems of boolean algebra.

7. Not So Negative (16 points)

For each of the following, translate the english statement into first-order logic, and then negate it. Your answer should push negation symbols as far in as possible; any negation symbol should be directly in front of a predicate. Don't forget to define predicates.

(a) [4 Points] Let the domain of discourse be all keys and all locks.

"Every bike lock has two different keys that open it."

(b) [4 Points] Let the domain of discourse be all solutions to problems.

"There is always a solution to every homework problem in CSE 311, but sometimes there is more than one solution."

For each of the following, negate the the first-order logic statement, and then translate it into English. Your answer should push negation symbols as far in as possible; any negation symbol should be directly in front of a predicate.

(c) [4 Points] Let the domain of discourse be all activities. Let $P(x, y)$ be "x is more fun than y".

$$\forall x \exists y (P(x, y) \rightarrow \neg P(y, x))$$

(d) [4 Points] Let the domain of discourse be courses at UW. Let $P(x, y)$ be "x is a pre-requisite for y". Let $Q(y)$ be "I am currently taking y".

$$\forall x \forall y ((P(x, y) \oplus Q(y)) \rightarrow \neg \exists z Q(z))$$

8. EXTRA CREDIT: Red Clubs (-NoValue- points)

There are many clubs/organizations that people at Ubiquity University (UU) can belong to. Some people at UU are students, some are TAs, and, following a famous Sherlock Holmes story, some people with red hair have banded together to form some specific clubs of their own. For this question, we will let the universe of discourse be all people at UU together with all clubs at UU and have predicates $\mathsf{Member}(x, c)$, to denote that x is a member of club c, and $\mathsf{Student}(x), \mathsf{TA}(x), \mathsf{Red}(x)$ to denote, respectively, that x is a student, is a TA, or has red hair. We also use predicates $\mathsf{Club}(c)$ to denote that c is a club and $\mathsf{Person}(x)$ to denote that x is a person. Using the following properties of UU:

- Property E_S: $\forall x\ (\mathsf{Member}(x, SR) \leftrightarrow (\mathsf{Student}(x) \wedge \mathsf{Red}(x)))$
- Property E_T: $\forall x\ (\mathsf{Member}(x, TR) \leftrightarrow (\mathsf{TA}(x) \wedge \mathsf{Red}(x)))$
- Property C: $\forall c\ (\mathsf{Club}(c) \rightarrow \exists d\ (\mathsf{Club}(d) \wedge \forall x\ (\mathsf{Person}(x) \rightarrow (\mathsf{Member}(x, d) \leftrightarrow \neg \mathsf{Member}(x, c)))))$
- Property G: $\forall c\ (\mathsf{Club}(c) \rightarrow \exists x\ (\mathsf{Person}(x) \wedge \neg \mathsf{Member}(x, c)))$

Explain precisely why there is at least one student at UU who does not have red hair.