CSE 311: Foundations of Computing I

Homework 1 (due Wednesday, October 1)

Directions: *Write up carefully argued solutions to the following problems. The first task is to be complete and correct. The more subtle task is to keep it simple and succinct. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use any results proven in lecture without proof. Anything else must be argued rigorously. Unless otherwise specified, all answers are expected to be given in closed form.*

0. The Tortoise, the Hare, and Floyd (15 points)

For each of the first three parts, translate the English statement into logical language.

(a) [4 Points] If the tortoise walks at a rate of one node per step, and the hare walks at a rate of two nodes per step, then the distance between them increases by one node per step.

(b) [4 Points] I can create an algorithm that detects if a linked list has a cycle if I know this idea.

(c) [4 Points] Either the tortoise and the hare will eventually be on the same node, which implies the linked list has a cycle, or an animal will reach the end of the list (which implies the linked list does not have a cycle).

In this part, we will give you a bad translation and ask you to explain why it doesn't work.

(d) [3 Points] Consider the following sentence:

> My code is wrong, because I didn't debug it.

If we let p be "my code is wrong", and we let q mean "I didn't debug it", a tempting translation for this sentence is $q \to p$. Explain (in no more than two sentences) why this translation isn't reasonable.

1. Nanananananananananagate! (12 points)

For each of the following parts, translate the English statement into logical language. Then, negate the statement using formal logic. Once you have negated the statement provide an easy to read English rendition of the sentence.

All formal logic sentences you provide should have any negation symbols immediately in front of propositional variables. For instance, $\neg(p \to q)$ is not simplified enough.

Your english sentences should avoid propositions like "Roger is not orange" where the word "not" appears.

(a) [4 Points] If x and y are the same, then x must be zero.

(b) [4 Points] 42 is a special number or Douglas Adams was wrong.

(c) [4 Points] **Note:** For this part, you may not use any negation symbols at all.

This course is the best one ever if and only if you are a student taking CSE 311.

2. The Curious Case of The Lying TAs (10 points)

A new UW CSE student wandered around the Paul Allen building on their first day in the major. They found (as many do) that there is a secret room in its basements. On the door of this secret room is a sign that says:

> `All ye who enter, beware! Every inhabitant of this room is either a TA who always lies or a student who always tells the truth!`

(a) [5 Points] The CSE student walked into the room, and two inhabitants walked up to the student. One of them said "at least one of us is a TA." Determine (with justification) all the possibilities for each of the two inhabitants.

(b) [5 Points] Three inhabitants walk up to the CSE student and surround the UW CSE student. One of them says "every TA in this circle has a TA to his immediate right." Determine (with justification) all the possibilities for each of the three inhabitants.

3. Contra-what now? (15 points)

Recall from lecture that if we have a statement $S := p \to q$, then

- the contrapositive of S is $\neg q \to \neg p$,
- the converse of S is $q \to p$, and
- the inverse of S is $\neg p \to \neg q$.

N.B.: While we will expect you to know what the contrapositive of a statement is, because it will prove to be very useful later on, the other two are less important; **we will not expect you to know converse or inverse by name**.

(a) [5 Points] Determine which of the following is true of the contrapositive of $S := p \to q$:

(i) The contrapositive of S is True precisely when S is True.

(ii) The contrapositive of S is False precisely when S is True.

(iii) When S is True, the contrapositive of S is sometimes True and sometimes False, depending on the statement S.

If your answer is (i) or (ii), make sure to provide two sentences (one where S is True and one where S is False, and explain how they back up your claim.

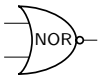
If your answer is (iii), make sure to provide two sentences (one where the contrapositive is True and one where it is False), and explain how they back up your claim.

(b) [5 Points] Answer the same question as in Part (a), but for *inverse* instead of contrapositive.

(c) [5 Points] Answer the same question as in Part (a), but for *converse* instead of contrapositive.

4. All You Need is Nor, Nor, Nor... (20 points)

The **NOR** connective takes two propositions and evaluates to True when both propositions are False and evaluates to False otherwise. In circuit diagrams, the gate for **NOR** is denoted by



The **NOR** of p and q is written as $p \downarrow q$. Demonstrate that we can construct *all the other connectives* by just using **NOR** by writing propositional formulae for each of the following while *only using **NOR** connectives*:

Hint: It's okay to use a single input/output more than once.

(a) [5 Points] $\neg p$

(b) [5 Points] $p \vee q$

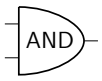
(c) [5 Points] $p \wedge q$

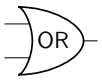
(d) [5 Points] $p \leftrightarrow q$

5. Mo Gates, Mo Problems! (10 points)

Using only...

AND Gates, **OR** Gates, and Inverters (**NOT** Gates),





draw the diagram of a circuit with **three** inputs that computes the function $M(p, q, r)$, where the following define M:

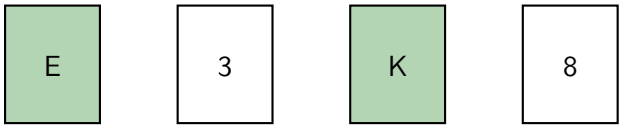
$$M(\mathsf{T}, q, r) \coloneqq q$$
$$M(\mathsf{F}, q, r) \coloneqq r$$

6. The Majority Wins! (10 points)

Find a compound proposition involving the propositional variables p, q, and r that is true precisely when a majority of p, q, and r are true. Explain why your answer works.

7. Card iff Alcohol? (8 points)

(a) [4 Points] You are presented with four *two-sided* (one green, one white) cards:



On the green side of each card is a letter, and on the white side is a number.

Consider the following rule:

> If a card has a vowel on one side, then it has an even number on the other side.

Which cards would need to be turned over to check if the rule is true? Explain your answer in a few sentences.

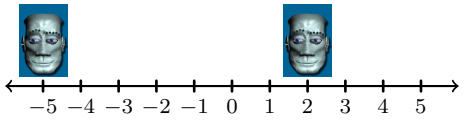
(b) [4 Points] The manager of a local club suspects that the bartender (the only one who gives both non-alcoholic and alcoholic drinks in the bar) isn't checking IDs correctly. For simplicity, assume everyone in the bar is either drinking coke or beer. Which people do you need to talk with to make sure she hasn't broken the law?: the ones with coke? beer? the ones under 21? over?
Hint: It might be useful to look over this question after you've solved both parts.

8. EXTRA CREDIT: XOR, I hardly... (-NoValue- points)

You are given two memory registers, each with the same number of bits. You have an operation, $\texttt{XOR}(R_1, R_2)$, which takes two registers, R_1 and R_2, performs bitwise $\oplus$ between them, and stores the result in R_1.

Show how you can swap the contents of the two registers using a sequence of XORs without temporary memory registers.

9. EXTRA CREDIT: Robot Rumble (-NoValue- points)



Two robots are pushed out of an airplane onto one of the numbers on an infinite number line. Each robot leaves its parachute where it lands and starts executing its program. Your task is to write a program to be given to both robots that will always make them collide. Your program will be a string using only the following four commands:

- `L`: go left one unit
- `R`: go right one unit
- `S`: skip the next command, unless I'm on a parachute
- `0-9`: Go to the nth command

Some more details:

- The program starts running from the 0th character.
- Your program may be at most 10 characters long.
- The robots each run one command at a time, simultaneously.
- Each command takes the same amount of time to run.
- The capitalization of the commands is important.

To submit this question, do the following two things:

(1) Submit your *answer* using the check script at: `http://www.countablethoughts.com/perfectly/symmetrical/violence/never/solves/anything/now.now.py`

(2) In your normal homework write-up, explain in a couple sentences how your answer works. For the TAs' sanity, please repeat your answer in your description.