CSE 311: Foundations of Computing I — Autumn 2014

Practice Final: Section X — YY ZZ

Name: ______________________________________

UW ID: ______________________________________

Instructions:

- Closed book, closed notes, no cell phones, no calculators.
- You have **110 minutes** to complete the exam.
- Answer all problems on the exam paper.
- If you need extra space use the back of a page.
- Problems are not of equal difficulty; if you get stuck on a problem, move on.
- You may tear off the last two pages of equivalence and inference rules. These must be handed in at the end but will not be graded.
- It may be to your advantage to read all the problems before beginning the exam.

Score Table Here

1. [? points]
Let $\Sigma = \{0, 1\}$. Prove that the language $L = \{x \in \Sigma^* : \#_0(x) < \#_1(x)\}$ is irregular.

2. [? points]
Define

$$T(n) = \begin{cases} n & \text{if } n = 0, 1 \\ 4T\left(\lfloor \frac{n}{2} \rfloor\right) + n & \text{otherwise} \end{cases}$$

Prove that $T(n) \leq n^3$ for $n \geq 3$.

3. [? points]
Let $\Sigma = \{0, 1, 2\}$.
Consider $L = \{w \in \Sigma^*$: Every 1 in the string has at least one 0 before and after it$\}$.

a) Give a regular expression that represents A.

b) Give a DFA that recognizes A.

c) Give a CFG that generates A.

4. [? points]
Consider the following CFG: **S** $\rightarrow$ **SS** | **S**1 | **S**01. Another way of writing the recursive definition of this set, Q, is as follows:

- $\varepsilon \in Q$
- If $S \in Q$, then $S1 \in Q$ and $S01 \in Q$
- If $S, T \in Q$, then $ST \in Q$.

Prove, by structural induction that if $w \in Q$, then w has at least as many 1's as 0's.

5. [? points]
For each of the following answer True or False and give a short explanation of your answer.

- Any subset of a regular language is also regular.

- The set of programs that loop forever on at least one input is decidable.

- If $\mathbb{R} \subseteq A$ for some set A, then A is uncountable.

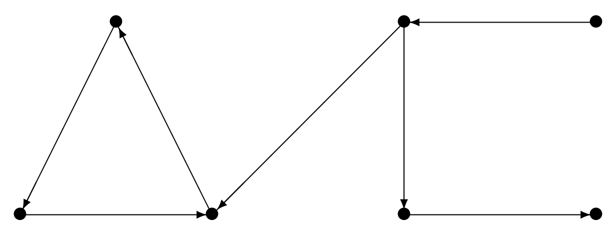
- If the domain of discourse is people, the logical statement

  $$\exists x \ (P(x) \rightarrow \forall y \ (x \neq y \rightarrow \neg P(y))$$

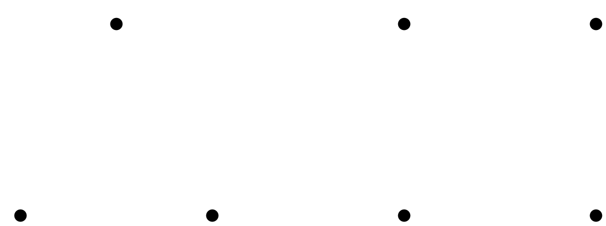
  can be correctly translated as "There exists a unique person who has property P".

- $\exists x \ (\forall y \ P(x, y)) \rightarrow \forall y \ (\exists x \ P(x, y))$ is true regardless of what predicate P is.

6. [? points]
The following is the graph of a binary relation R.

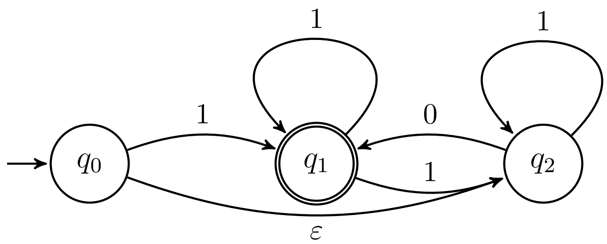


a) Draw the transitive-reflexive closure of R.



b) Let $S = \{(X, Y) : X, Y \in \mathcal{P}(\mathbb{N}) \wedge X \subseteq Y\}$.
Recall that R is antisymmetric iff $((a, b) \in R \wedge a \neq b) \rightarrow (b, a) \notin R$.
Prove that S is antisymmetric.

7. [? points]
Convert the following NFA into a DFA using the algorithm from lecture.



8. [? points]
Let $\Sigma = \{0, 1, 2\}$. Construct a DFA that recognizes exactly strings with a 0 in all positions i where i mod $3 = 0$.