

CSE 311 Foundations of Computing I

Lecture 20
Relations, Directed Graphs
Spring 2013

1

Announcements

- Reading assignments
 - 7th Edition, Section 9.1 and pp. 594-601
 - 6th Edition, Section 8.1 and pp. 541-548
- Upcoming topic
 - Finite State Machines

2

Definition of Relations

Let A and B be sets,
A **binary relation from A to B** is a subset of $A \times B$

Let A be a set,
A **binary relation on A** is a subset of $A \times A$

3

Relation Examples

$$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$$

$$R_2 = \{(x, y) \mid x \equiv y \pmod{5}\}$$

$$R_3 = \{(c_1, c_2) \mid c_1 \text{ is a prerequisite of } c_2\}$$

$$R_4 = \{(s, c) \mid \text{student } s \text{ had taken course } c\}$$

4

Properties of Relations

Let R be a relation on A

R is **reflexive** iff $(a,a) \in R$ for every $a \in A$

R is **symmetric** iff $(a,b) \in R$ implies $(b,a) \in R$

R is **antisymmetric** iff $(a,b) \in R$ and $a \neq b$ implies $(b,a) \notin R$

R is **transitive** iff $(a,b) \in R$ and $(b,c) \in R$ implies $(a,c) \in R$

Combining Relations

Let R be a relation from A to B

Let S be a relation from B to C

The composite of R and S, $S \circ R$ is the relation from A to C defined

$$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

Examples

$(a,b) \in \text{Parent}$: b is a parent of a

$(a,b) \in \text{Sister}$: b is a sister of a

What is $\text{Sister} \circ \text{Parent}$?

What is $\text{Parent} \circ \text{Sister}$?

$$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}$$

Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother, Husband, Wife express -

Uncle: b is an uncle of a

Cousin: b is a cousin of a

Powers of a Relation

$$R^2 = R \circ R = \{(a, c) \mid \exists b \text{ such that } (a,b) \in R \text{ and } (b,c) \in R\}$$

$$R^0 = \{(a,a) \mid a \in A\}$$

$$R^1 = R$$

$$R^{n+1} = R^n \circ R$$

9

Matrix representation

Relation R on $A = \{a_1, \dots, a_p\}$

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, a_j) \in R, \\ 0 & \text{if } (a_i, a_j) \notin R. \end{cases}$$

$\{(1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (3, 2), (3, 3), (4, 2), (4, 3)\}$

10

Directed graphs

$$G = (V, E)$$

V – vertices

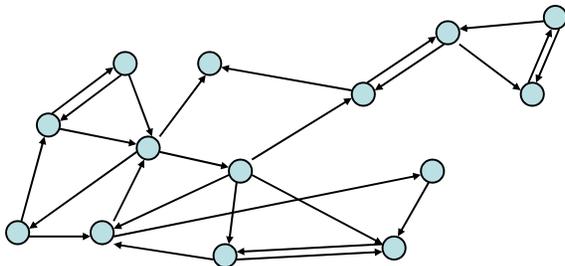
E – edges, order pairs of vertices

Path: v_1, v_2, \dots, v_k , with $(v_i, v_{i+1}) \in E$

Simple Path

Cycle

Simple Cycle

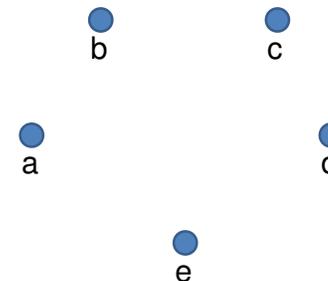


11

Representation of relations

Directed Graph Representation (Digraph)

$\{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e), (d, e)\}$



12

Computing the Composite Relation using the Graph Representation

If $S=\{(2,2),(2,3),(3,1)\}$ and $R=\{(1,2),(2,1),(1,3)\}$
compute $S \circ R$

13

Paths in relations

Let R be a relation on a set A . There is a path of length n from a to b if and only if $(a,b) \in R^n$

14

Connectivity relation

Let R be a relation on a set A . The connectivity relation R^* consists of the pairs (a,b) such that there is a path from a to b in R .

$$R^* = \bigcup_{k=0}^{\infty} R^k$$

Note: The text uses the wrong definition of this quantity. What the text defines (ignoring $k=0$) is usually called R^+

15

Properties of Relations

Let R be a relation on A

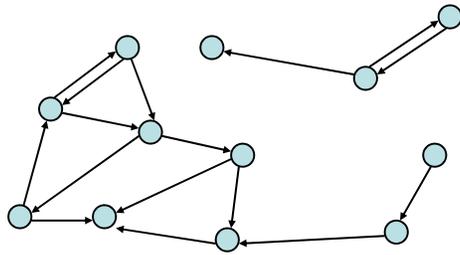
R is **reflexive** iff $(a,a) \in R$ for every $a \in A$

R is **symmetric** iff $(a,b) \in R$ implies $(b, a) \in R$

R is **transitive** iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

16

Transitive-Reflexive Closure



Add the minimum possible number of edges to make the relation transitive and reflexive

The transitive-reflexive closure of a relation R is the connectivity relation R^*

How is



related to



?

How is



related to



?

<http://genealogy.math.ndsu.nodak.edu/>

Mathematics Genealogy Project

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service of the [NDSU Department of Mathematics](#), in association with the [American Mathematical Society](#).

Please [email us](#) with feedback.

Edward Delano Lazowska

[MathSciNet](#)

Ph.D. [University of Toronto 1977](#)

Dissertation: *Characterizing Service Time and Response Time Distributions in Queueing Network Models of Computer Systems*

Advisor: [Kenneth Clem Sevcik](#)

Students:
Click [here](#) to see the students listed in chronological order.

Name	School	Year	Descendants
Thomas Anderson	University of Washington	1991	54
Robert Bedichek	University of Washington	1994	
John Bennett	University of Washington	1988	9
Brian Bershad	University of Washington	1990	16
Jeffrey Chase	University of Washington	1995	7
Sung Chung	University of Washington	1990	
Edward Felten	University of Washington	1993	8
Richard Garner	University of Washington	1982	
Patricia Jacobson	University of Washington	1984	
Henry (Hank) Levy	University of Washington	1981	123
Yi-Bing Lin	University of Washington	1990	13

20



Anderson



Mayr



Bauer



Caratheodory



Minkowski



Klein



Lipschitz



Dirichlet



Fourier



Lagrange



Euler



Johann Bernoulli



Jacob Bernoulli



Leibniz



Weigel



Rheticus



Copernicus



Beame



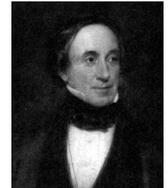
Cook



Quine



Whitehead



Hopkins



Sedgewick



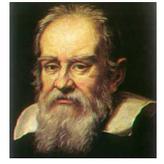
Smith



Newton



Barrow



Galileo

Nicolaus Copernicus
 Georg Rheticus
 Moritz Steinmetz
 Christoph Meurer
 Philipp Muller
 Erhard Weigel
 Gottfried Leibniz
 Noclac Malebranache
 Jacob Bernoulli
 Johann Bernoulli
 Leonhard Euler
 Joseph Lagrange
 Jean-Baptiste Fourier
 Gustav Dirichlet
 Rudolf Lipschitz
 Felix Klein
 C. L. Ferdinand Lindemann
 Herman Minkowski
 Constantin Carathéodory
 Georg Aumann
 Friedrich Bauer
 Manfred Paul
 Ernst Mayr
 Richard Anderson

Galileo Galilei
 Vincenzo Viviani
 Issac Barrow
 Isaac Newton
 Roger Cotes
 Robert Smith
 Walter Taylor
 Stephen Whisson
 Thomas Postlethwaite
 Thomas Jones
 Adam Sedgewick
 William Hopkins
 Edward Routh
 Alfred North Whitehead
 Willard Quine
 Hao Wang
 Stephen Cook
 Paul Beame

n-ary relations

Let A_1, A_2, \dots, A_n be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times \dots \times A_n$.

Relational databases

STUDENT

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Von Neuman	481080220	555	3.78
Russell	238082388	022	3.85
Einstein	238001920	022	2.11
Newton	1727017	333	3.61
Karp	348882811	022	3.98
Bernoulli	2921938	022	3.21

Relational databases

STUDENT

Student_Name	ID_Number	Office	GPA	Course
Knuth	328012098	022	4.00	CSE311
Knuth	328012098	022	4.00	CSE351
Von Neuman	481080220	555	3.78	CSE311
Russell	238082388	022	3.85	CSE312
Russell	238082388	022	3.85	CSE344
Russell	238082388	022	3.85	CSE351
Newton	1727017	333	3.61	CSE312
Karp	348882811	022	3.98	CSE311
Karp	348882811	022	3.98	CSE312
Karp	348882811	022	3.98	CSE344
Karp	348882811	022	3.98	CSE351
Bernoulli	2921938	022	3.21	CSE351

What's not so nice?

Relational databases

STUDENT

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Von Neuman	481080220	555	3.78
Russell	238082388	022	3.85
Einstein	238001920	022	2.11
Newton	1727017	333	3.61
Karp	348882811	022	3.98
Bernoulli	2921938	022	3.21

TAKES

ID_Number	Course
328012098	CSE311
328012098	CSE351
481080220	CSE311
238082388	CSE312
238082388	CSE344
238082388	CSE351
1727017	CSE312
348882811	CSE311
348882811	CSE312
348882811	CSE344
348882811	CSE351
2921938	CSE351

Better

Database Operations: Projection

Find all offices: $\Pi_{\text{Office}}(\text{STUDENT})$

Office
022
555
333

Find offices and GPAs: $\Pi_{\text{Office,GPA}}(\text{STUDENT})$

Office	GPA
022	4.00
555	3.78
022	3.85
022	2.11
333	3.61
022	3.98
022	3.21

Database Operations: Selection

Find students with GPA > 3.9 : $\sigma_{\text{GPA}>3.9}(\text{STUDENT})$

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Karp	348882811	022	3.98

Retrieve the name and GPA for students with GPA > 3.9 :

$\Pi_{\text{Student_Name}, \text{GPA}}(\sigma_{\text{GPA}>3.9}(\text{STUDENT}))$

Student_Name	GPA
Knuth	4.00
Karp	3.98

Database Operations: Natural Join

Student \bowtie Takes

Student_Name	ID_Number	Office	GPA	Course
Knuth	328012098	022	4.00	CSE311
Knuth	328012098	022	4.00	CSE351
Von Neuman	481080220	555	3.78	CSE311
Russell	238082388	022	3.85	CSE312
Russell	238082388	022	3.85	CSE344
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Newton	1727017	333	3.61	CSE312
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Karp	348882811	022	3.98	CSE312
Karp	348882811	022	3.98	CSE344
Karp	348882811	022	3.98	CSE351
Bernoulli	2921938	022	3.21	CSE351