University of Washington
Department of Computer Science and Engineering
CSE 311, Spring 2013

May 22, 2013

Homework 7, Due Wednesday, May 29, 2013

Problem 1:

A relation R is called *circular* iff $(c, a) \in R$ whenever $(a, b) \in R$ and $(b, c) \in R$. Prove that for any reflexive relation R, R is circular if and only if R is both symmetric and transitive.

Problem 2:

Let R be the relation on pairs of positive integers, $\mathbb{Z}^+ \times \mathbb{Z}^+$ given by $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Prove that R is reflexive, symmetric and transitive.

Problem 3:

A directed graph is called *acyclic* iff if it does not contain a directed cycle (a non-empty path from a vertex to itself). Show that for every directed acyclic graph G, the transitive-reflexive closure of the relation R represented by G is antisymmetric.

Problem 4:

Is the transitive-reflexive closure of R always equal to the transitive-reflexive closure of R^2? Justify your answer.

Problem 5:

Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram, document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.

a) The set of all binary strings that start with 0 and have even length, or start with 1 and have odd length.

b) The set of all binary strings that have a 1 in every even-numbered position counting from the start of the string with the start of the string counting as position 1.

Problem 6:

Give state diagrams for (deterministic) finite state machines that recognize each of the following sets of strings. Indicate the start and final states in your diagrams and clearly label each state. In addition to the diagram. document each design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.

a)
   i. The set of all binary strings that contain at least two 1's.
   ii. The set of all binary strings that contain at most two 0's. Use different state labels from the ones you used for part i.
   iii. Combine the machines from parts i. and ii. to produce a machine that recognizes the set of all binary strings that contain at least two 1's and at most two 0's.

b) The set of all binary strings that don't contain 001.

Problem 7:

Design a finite state machine with outputs for a Candy Machine that dispenses a Gumball for 10 cents or M&M's for 15 cents. The machine takes nickels and dimes. It returns change if too much money is inserted or if the cost of the item selected is less than the amount of money deposited. A state is allowed to have multiple outputs.

Extra Credit 8:

Give a state diagram for a (deterministic) finite state machine that recognizes the set of all binary strings that represent integers that are multiples of 3 when read from left to right. Indicate the start and final states in your diagram and clearly label each state. In addition to the diagram document your design by writing a phrase for each state describing the set of inputs that lead from the start state to that state.