University of Washington May 2, 2013
Department of Computer Science and Engineering
CSE 311, Spring 2013

Homework 5, Due Wednesday, May 8, 2013

Problem 1:

Compute the GCD of 91 and 434 using the Euclidean Algorithm. Show the intermediate values that are computed.

Problem 2:

Use the Euclidean algorithm to solve the following problems:
a) Find an inverse of 4 modulo 21.
b) Find an inverse of 5 modulo 18.
c) Solve $13x \equiv 7 \pmod{56}$ for x.

Problem 3:

Prove that for every integer n, there are n consecutive composite integers. [Hint: Consider the n consecutive integers starting with $(n+1)! + 2$.]

Problem 4:

Prove that for every positive integer n,

$$\sum_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2.$$

Problem 5:

Prove that 3 divides $n^3 + 2n$ when n is a positive integer.

Problem 6:

Let x be any fixed real number with $x > -1$. Prove that $(1+x)^n \geq 1 + nx$ for every integer $n \geq 0$.

Problem 7:

Let f_n be the n-th Fibonacci number where $f_0 = 0$, $f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Prove that

$$f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$$

for every positive integer n.

Extra Credit 8:

Two integers a and b are *relatively prime* if and only if $gcd(a, b) = 1$. Consider any $n + 1$ numbers between 1 and $2n$ (inclusive). Show that some pair of them are relatively prime.