University of Washington
Department of Computer Science and Engineering
CSE 311, Spring 2013

April 27, 2013

Homework 4, Due Wednesday, May 1, 2013

Problem 1:

Suppose that the sets A, B, and C have 4, 6, and 8 elements respectively. For each of the statements below, indicate whether it is certainly true, or certainly false, or can be either true or false. Briefly justify your answers:
(a) $A \cup B$ has exactly 10 elements.
(b) $A \cap B$ has at most 4 elements.
(c) If $A \cup B$ has m elements and $A \cap B$ has n elements, then $m + n$ is always equal to 10.
(d) $A \cup B$ has at most as many elements as $A \cup C$.
(e) $(A \oplus B) \oplus (B \oplus C) \oplus (A \oplus B \oplus C)$ has exactly 6 elements; $\oplus$ denotes the symmetric difference of two sets.

Problem 2:

Consider the following functions from the positive integer numbers (including zero) to the positive integers (including zero). For each of the functions below, indicate the following: (i) its domain, (ii) its range, (iii) whether the function is one-to-one, (iv) whether the function is onto. Briefly justify your answers.
(a) $f(n) = n + 1$ if n is even, and $f(n) = n - 1$ if n is odd.
(b) $f(n) = n/2$ if n is even, $f(n) = (3n + 1)/2$ if n is odd.
(c) $f(n) = 2^n$.
(d) $f(n) = \lfloor \log(n) \rfloor$. The logarithm is in base two. If x is a real number, then $\lfloor x \rfloor$ denotes the largest integer less than or equal to x; for example $\lfloor 4.7 \rfloor = 4$ and $\lfloor 4.0 \rfloor = 4$.
(e) $f(n) = \left\lfloor \frac{10n-1}{n-1} \right\rfloor$.

Problem 3:

Prove that if n is an integer then $n^2 \bmod 5$ is either 0, 1, or 4.

Problem 4:

Let a, b be integers and c, n be positive integers.
Prove that if $a \equiv b \pmod{n}$ then $ac \equiv bc \pmod{cn}$.

Problem 5:

Show that a positive integer is divisible by 11 if and only if the difference of the sum of its decimal digits in even-numbered positions and the sum of the decimal digits in its odd-numbered positions is divisible by 11.

Problem 6:

For each $a \in \{1, \ldots, 12\}$ determine the smallest integer $k \geq 1$ such that $a^k \bmod 13 = 1$.

Problem 7:

Compute $43^{148} \bmod 1000$ using the fast modular exponentiation algorithm. Show your intermediate results. (Hint: this problem only requires 9 multiplications.)

Extra Credit 8:

The number 2011 is a prime number.

1. Compute $3 \times 1341 \bmod 2011$ (you can use a calculator).
2. Compute $4 \times 503 \bmod 2011$ (you can use a calculator).
3. Compute $5 \times 1609 \bmod 2011$ (you can use a calculator).
4. Compute $2010! \bmod 2011$; this is the same as $1 \times 2 \times \cdots \times 2010 \bmod 2011$ (you don't need a calculator here).