University of Washington — April 15, 2013
Department of Computer Science and Engineering
CSE 311, Spring 2013

Homework 2, Due Wednesday, April 17, 2013

Problem 1:
A standard notation for the integer represented by a sequence of bits is to surround the bits by $(\quad)_2$. For example, $(11)_2 = 3$ and $(0101)_2 = 5$. In this question you will construct a circuit that takes two two-bit integers $(x_1x_0)_2$ and $(y_1y_0)_2$ and computes the four output bits for their integer product $(z_3z_2z_1z_0)_2$.
a) Give sum-of-products forms for the two output bits of the product, $(a_1a_0)_2$, of $(x_1x_0)_2$ and $(y_0)_2$. Do the same for $(x_1x_0)_2$ multiplied by $(y_1)_2$ yielding $(b_2b_1)_2$. These are the bits produced as part of applying the usual elementary school method to multiply numbers.
b) Use the minimized sum-of-products forms for one-bit adders given in class, together with the results of the above two products to produce sum-of-products forms for z_3, z_2, z_1, z_0. Some of the inputs you give to the one-bit adders may be constants. Use Boolean algebra to minimize the resulting sum-of-products forms using only x_1, x_0, y_1, y_0.
c) Draw circuit diagrams for the results.

Problem 2:
Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives in two ways for each part. For (i) let the domain consist of the students in your class and for (ii) let the domain consist of all people.
a) All students in your class can solve quadratic equations.
b) Some student in your class does not want to be rich.

Problem 3:
Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.
a) $\exists x(R(x) \wedge H(x))$
b) $\forall x(R(x) \rightarrow H(x))$
c) $\forall x(R(x) \wedge H(x))$
d) $\exists x(R(x) \rightarrow H(x))$

Problem 4-1:
Let $F(x, y)$ be the statement "x can fool y," where the domain for both x and y constis of all people in the world. Use quantifiers to express each of these statements:
a) Everybody can fool Fred.
b) Everybody can fool somebody.
c) Jane can fool exactly one person.
d) No one can fool himself or herself.

Problem 4-2:

Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.
a) $\exists x \forall y(x + y = y)$
b) $\forall x \forall y(((x \neq 0) \wedge (y \neq 0)) \leftrightarrow (x\ y \neq 0))$

Problem 5:

Rewrite each of these statements so that the negations appear only next to predicate symbols (that is, so that no negation is outside a quantifier or an expression involving logical connectives).
a) $\neg \exists y \exists x P(x, y)$
b) $\neg \exists y(Q(y) \wedge \forall x \neg R(x, y))$
c) $\neg \exists y(\exists x R(x, y) \vee \forall x S(x, y))$

Problem 6:

Give examples of predicates and domains to show that the statements $\forall x(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are not logically equivalent.

Extra Credit 7:

Sometimes there are logical inference tasks where possibilities are mutually exclusive and we can express solutions using a simple assignment table rather than separate predicates. Consider the following:
The popularity of a certain well-known junior wizard has led to a huge increase in the number of academies offering magic courses to would-be sorcerers. Each academy specializes in its own area of magic. Below are the details of five lucky candidates who have won places at different academies: From the information given, work out at which establishment each is a pupil, their main area of study, and the Professor teaching them that subject?

1. Larry Lester is a pupil at Porkwens Academy, but not to study invisibility.
2. Professor Deerkey teaches wandwork, but not at Porkwens or Hamboils.
3. Lottie Baxter is not at BoarPoack, and isn't being taught by Professor McTavish, who is not a specialist in flying.
4. Barry Carter is taught by Professor Tumbledown.
5. Carrie Foster is studying potions.
6. One of the five is attending Sowrash Academy to study fortune-telling.
7. Professor Squirrel teaches at Gruntpimples Academy.
8. Gary Dexter is the other student and Professor Snoop is one of the five instructors.

You do **not** need to show your reasoning.