CSE 311: Foundations of Computing I
Assignment #1
April 3, 2013
Due: Wednesday, April 10, 2013

Reading Assignment: Read Sections 1.1-1.3 of 7th edition (1.1-1.2 of 6th edition) of the text. (make sure that you understand the examples).

Problems:

1. Write each of these statements in the form “if p, then q in English. [Hint: refer to the list of common ways to expression conditional statements provide in 1.1 of the text.]

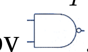
   (a) To be a citizen of this country, it is sufficient that you were born in the United States.

   (b) The Mariners will win the World Series unless the Blue Jays win it.

   (c) The beach erodes whenever there is a storm.

   (d) I will send you the package if you send me your address.

2. The NAND connective takes two propositions and evaluates to false when both propositions are true and evaluates to true otherwise. NAND of p and q is written as $p|q$.

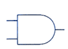
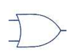
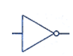
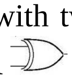
   (In circuit diagrams the gate for NAND is denoted by .)
   Show how to write propositional formulas *using the NAND connective and the variables* p *and* q *but no constants or other connectives* that are equivalent to each of the following:

   (a) $\neg p$

   (b) $p \vee q$

   (c) $p \wedge q$

   (d) $p \rightarrow q$

3. Using only AND  gates, OR  gates. and inverters (NOT  gates), draw the diagram of a circuit with two inputs that computes the same function of its inputs that a single two-input XOR  gate does.

4. State in English the converse and contrapositive of each of the following implications:

   (a) If a is pushed onto the stack before b, then b is popped before a.

   (b) If the input is correct and the program terminates, then the output is correct. (Be sure to use De Morgan’s Law to simplify the contrapositive so that the statement reads more naturally in English.)

5. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.

6. The following two statements form the basis of the most important methods for automated theorem proving. Use truth tables to prove that they are tautologies (i.e., that they always evaluate to true).

   (a) Resolution: $((p \vee q) \wedge (\neg q \vee r)) \rightarrow (p \vee r)$

   (b) Modus ponens: $((p \wedge (p \rightarrow q)) \rightarrow q$

7. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent using equivalences.

8. **Extra Credit:** You have two memory registers, each with the same number of bits. You have an operation, `XOR(R1,R2)`, which takes two registers, `R1` and `R2`, performs bitwise $\oplus$ between them, and stores the result in `R1`. Show how you can swap the contents of the two registers using a sequence of `XOR`s without temporary memory registers. Explain why this works.