CSE 311: Foundations of Computing

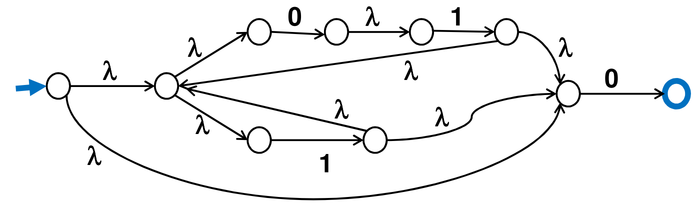
Fall 2013

Lecture 25: Non-regularity and limits of FSMs

highlights

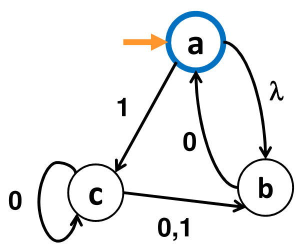
NFAs from Regular Expressions

$(01 \cup 1)^*0$

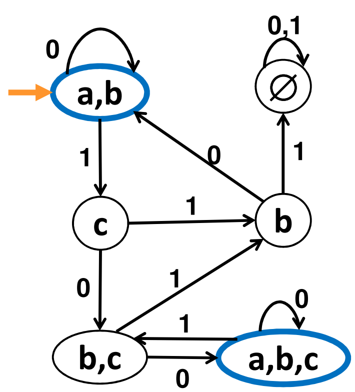


highlights

"Subset construction": NFA to DFA

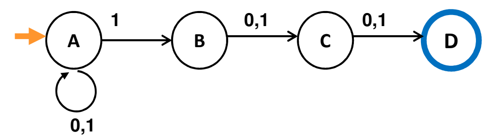


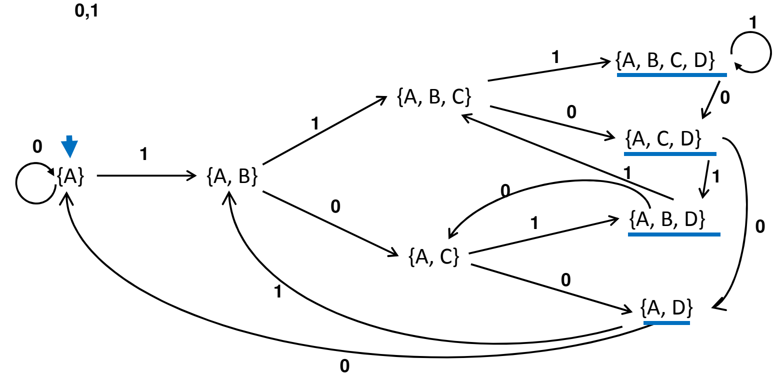
NFA



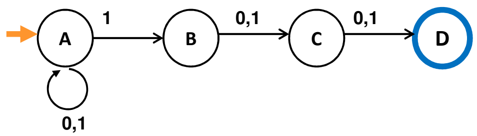
DFA

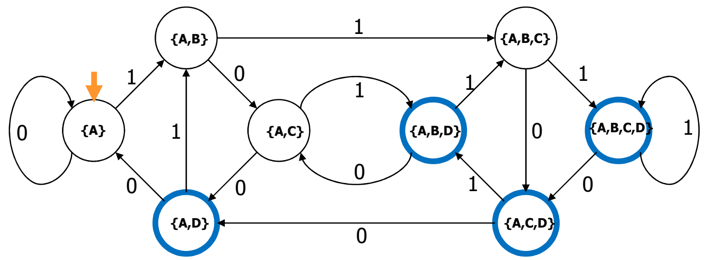
1 in third position from end





redrawing





DFAs ≡ regular expressions

We have shown how to build an optimal DFA for every regular expression

- Build NFA
- Convert NFA to DFA using subset construction
- Minimize resulting DFA

Theorem: A language is recognized by a DFA if and only if it has a regular expression

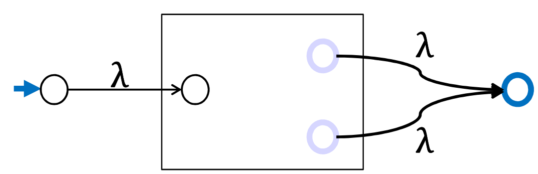
generalized NFAs

- Like NFAs but allow
  - Parallel edges
  - Regular Expressions as edge labels

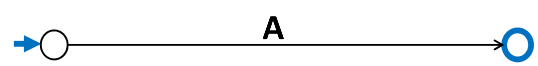
    NFAs already have edges labeled λ or ***a***
- An edge labeled by A can be followed by reading a string of input chars that is in the language represented by A
- A string x is accepted iff there is a path from start to final state labeled by a regular expression whose language contains x

starting from an NFA

Add new start state and final state



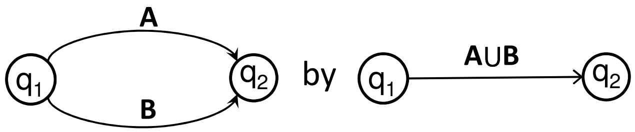
Then eliminate original states one by one, keeping the same language, until it looks like:



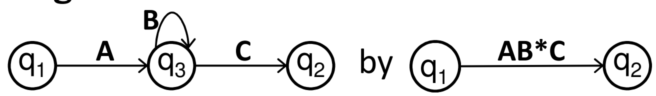
Final regular expression will be A

only two simplification rules

- **Rule 1**: For any two states q_1 and q_2 with parallel edges (possibly q_1=q_2), replace



- **Rule 2**: Eliminate non-start/final state q_3 by replacing all

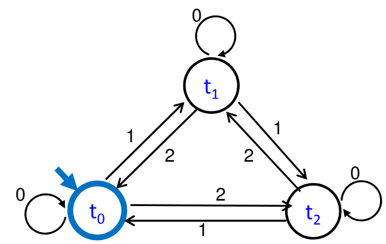


for *every* pair of states q_1, q_2 (even if q_1=q_2)

converting an NFA to a regular expression

Consider the DFA for the mod 3 sum

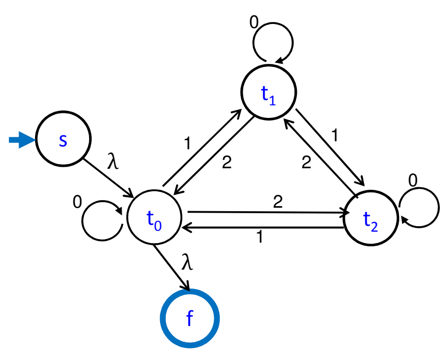
– Accept strings from {0,1,2}* where the digits mod 3 sum of the digits is 0



splicing out a node

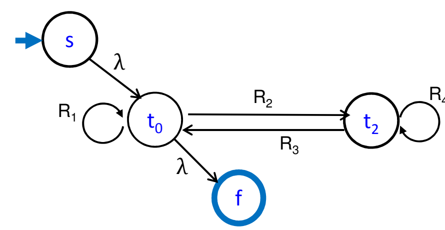
Label edges with regular expressions

$t_0 \to t_1 \to t_0$: 10*2
$t_0 \to t_1 \to t_2$: 10*1
$t_2 \to t_1 \to t_0$: 20*2
$t_2 \to t_1 \to t_2$: 20*1

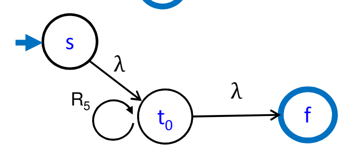


finite automaton without t_1

R_1: 0 ∪ 10*2
R_2: 2 ∪ 10*1
R_3: 1 ∪ 20*2
R_4: 0 ∪ 20*1



R_5: $R_1 \cup R_2R_4{}^*R_3$



Final regular expression:
(0 ∪ 10*2 ∪ (2 ∪ 10*1)(0 ∪ 20*1)*(1 ∪ 20*2))*

what can finite state machines do?

- We've seen how we can get DFAs to recognize all regular languages

- What about some other languages we can generate with CFGs?
  - { $0^n 1^n : n \geq 0$ } ?
  - binary palindromes?
  - strings of balanced parentheses?

A=$\{0^n 1^n : n \geq 0\}$ cannot be recognized by any DFA

Consider the infinite set of strings

S={λ, 0, 00, 000, 0000, ...}

Claim: No two strings in S can end at the same state of any DFA for A

Proof:

Suppose n≠m and 0^n and 0^m end at the same state p of some DFA for A.
Since 0^n1^n is in A, following 1^n after state p must lead to a final state.

But then the DFA would also accept 0^m1^n
which is a contradiction to the DFA recognizing A.

Given claim, the # of states of any DFA for A must be ≥ |S| which is not finite, which is impossible for a DFA.

B = {binary palindromes} can't be recognized by any DFA

Consider the infinite set of strings

S={λ, 0, 00, 000, 0000, ...}={0^n : n ≥ 0}

Claim: No two strings in S can end at the same state of any DFA for B

Proof:

Suppose n≠m and 0^n and 0^m end at the same state p of some DFA for B.
Since 0^n10^n is in B, following 10^n after state p must lead to a final state.

But then the DFA would also accept 0^m10^n which is not in B
and is a contradiction since the DFA recognizes B.

Given claim, the # of states of any DFA for B must be ≥ |S| which is not finite, which is impossible for a DFA.

general: how to show language L has no DFA

- Find a "hard" infinite set S={s_0,s_1,...,s_n,...} of strings that might be prefixes of strings in L

- Show that S is hard by showing that no two strings s_n≠s_m in S can end at the same state of any DFA recognizing L
  - For each pair s_n≠s_m find an extender string t depending on n,m so that exactly one of s_nt and s_mt is in L

- Conclude that any DFA for L would need ≥ |S| states which is not finite, and so impossible

P = {strings of balanced parentheses}

pattern matching

- Given
  - a string, **s**, of **n** characters
  - a pattern, **p**, of **m** characters
  - usually **m<<n**
- Find
  - all occurrences of the pattern **p** in the string **s**

- Obvious algorithm:
  - try to see if **p** matches at each of the positions in **s**
    stop at a failed match and try the next position

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

pattern **p** = x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
x y x y
x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
x y x y
x
x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
x y x y
x
x y
x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
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x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
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string **s** = x y x x y x y x y y x y x y x y y x y x y x x
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x y x y y x y x y x x

String **s** = x y x x y x y x y y x y x y x y y x y x y x x
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x y x y y
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x
x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x
x y x y
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x y x y y
x
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x
x y x
x y x y y x y x y x x

string **s** = x y x x y x y x y y x y x y x y y x y x y x x

```
x y x y
  x
    x y
      x y x y y
        x
          x y x y y x y x y x x
            x
              x y x
                x
                  x y x y y x y x y x x
```


string **s** = x y x x y x y x y y x y x y x y y x y x y x x

```
x y x y
  x
    x y
      x y x y y
        x
          x y x y y x y x y x x
            x
              x y x
                x
                  x
                    x y x y y x y x y x x
```


string **s** = x y x x y x y x y y x y x y x y y x y x y x x

```
x y x y
  x
    x y
      x y x y y
        x
          x y x y y x y x y x x
            x
              x y x
                x
                  x
                    x y x y y
                      x y x y y x y x y x x
```


string **s** = x y x x y x y x y y x y x y x y y x y x y x x

```
x y x y
  x
    x y
      x y x y y
        x
          x y x y y x y x y x x
            x
              x y x
                x
                  x
                    x y x y y
                      x
                        x y x y y x y x y x x
```

Worst-case time O(**mn**)

better pattern matching via finite automata

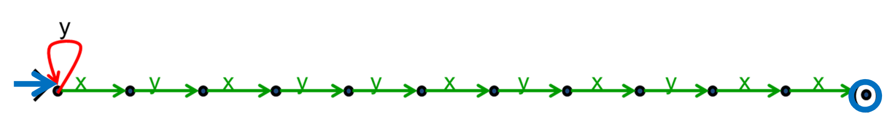
- Build a DFA for the pattern (preprocessing) of size O(m)
  - Keep track of the 'longest match currently active'
  - The DFA will have only **m+1** states
- Run the DFA on the string **n** steps
- Obvious construction method for DFA will be $O(m^2)$ but can be done in O(m) time.
- Total O(m+n) time

building a DFA for the pattern

pattern **p**=x y x y y x y x y x x

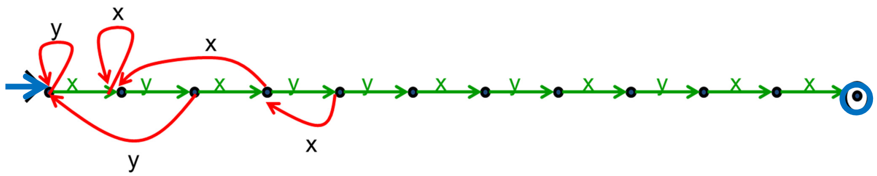
preprocessing the pattern

pattern **p**=x y x y y x y x y x x



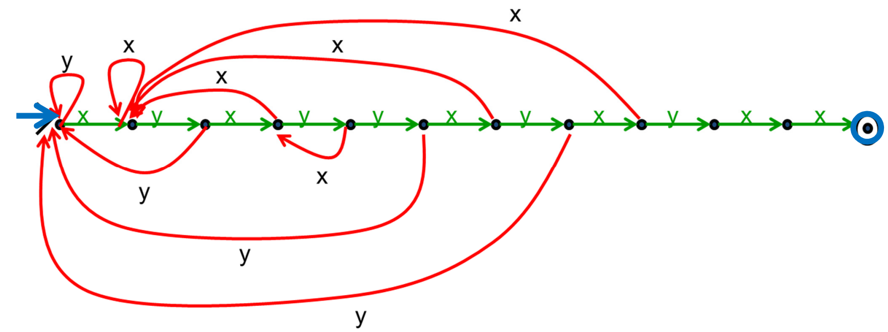
preprocessing the pattern

pattern **p**=x y x y y x y x y x x



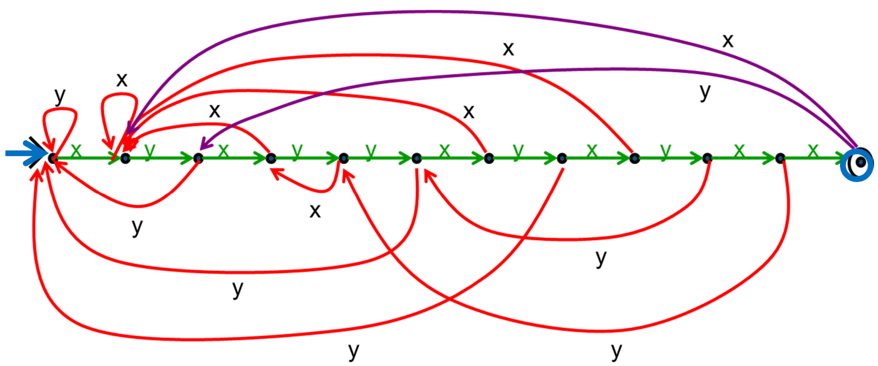
preprocessing the pattern

pattern **p**=x y x y y x y x y x x



preprocessing the pattern

pattern **p**=x y x y y x y x y x x



generalizing

- **Can search for arbitrary combinations of patterns**
  - **Not just a single pattern**
  - **Build NFA for pattern then convert to DFA 'on the fly'.**

    Compare DFA constructed above with subset construction for the obvious NFA.