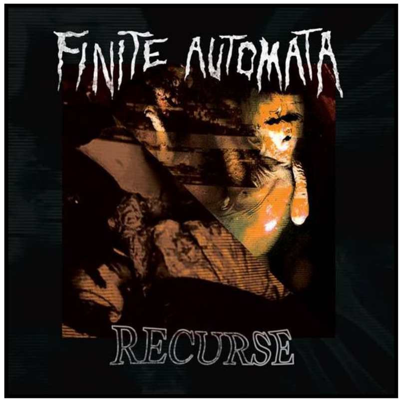
CSE 311: Foundations of Computing

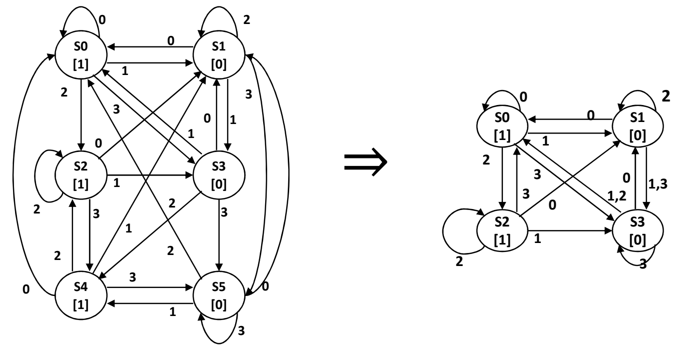
Fall 2013

Lecture 24: NFAs, regular expressions, and NFA→DFA



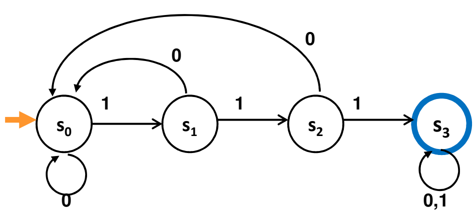
highlights

- **FSMs with output at states**
- **State minimization**



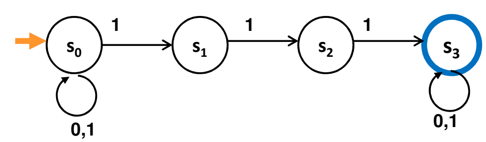
highlights

Lemma: The language recognized by a DFA is the set of strings x that label some path from its start state to one of its final states



nondeterministic finite automaton (NFA)

- **Graph with start state, final states, edges labeled by symbols (like DFA) but**
  - Not required to have exactly 1 edge out of each state labeled by each symbol— can have 0 or >1
  - Also can have edges labeled by empty string λ
- **Definition: x is in the language recognized by an NFA if and only if x labels a path from the start state to some final state**



three ways of thinking about NFAs

- Outside observer: Is there a path labeled by x from the start state to some final state?
- Perfect guesser: The NFA has input x and whenever there is a choice of what to do it magically guesses a good one (if one exists)
- Parallel exploration: The NFA computation runs all possible computations on x step-by-step at the same time in parallel

NFAs and regular expressions

Theorem: For any set of strings (language) A described by a regular expression, there is an NFA that recognizes A.

Proof idea: Structural induction based on the recursive definition of regular expressions...

Note: One can also find a regular expression to describe the language recognized by any NFA but we won't prove that fact

regular expressions over Σ

- **Basis:**
  - $\varnothing$, λ are regular expressions
  - *a* is a regular expression for any $a \in \Sigma$
- **Recursive step:**
  - If A and B are regular expressions then so are:

    $(A \cup B)$

    (AB)

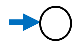
    A^*

basis

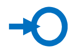
- Case $\varnothing$:
- Case λ:
- Case *a*:

basis

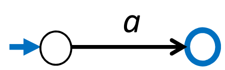
- Case ∅:



- Case λ:

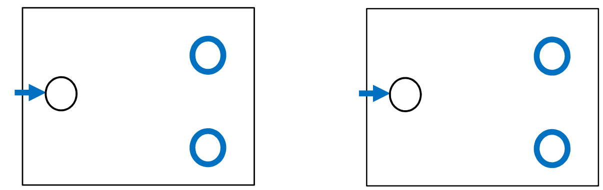


- Case *a*:



inductive hypothesis

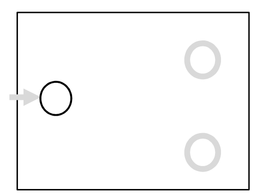
- Suppose that for some regular expressions $\boldsymbol{A}$ and $\boldsymbol{B}$ there exist NFAs N_A and N_B such that N_A recognizes the language given by $\boldsymbol{A}$ and N_B recognizes the language given by $\boldsymbol{B}$



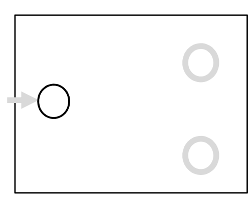
N_A N_B

inductive step

Case (**A** ∪ **B**):



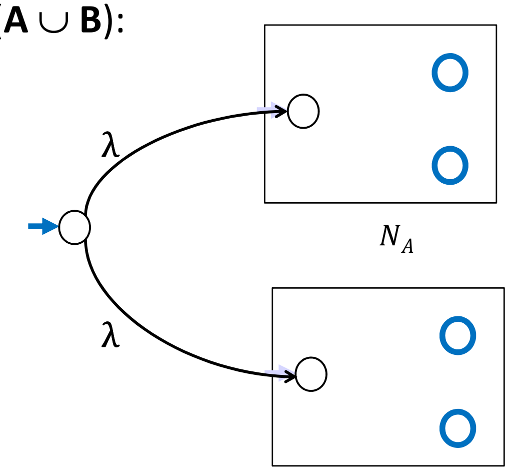
N_A



N_B

inductive step

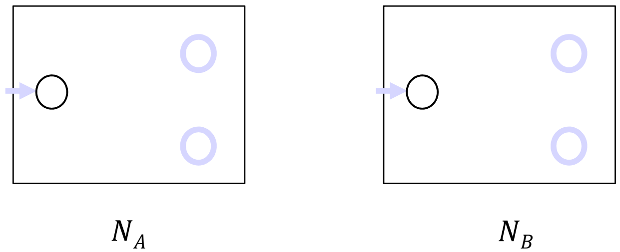
Case (**A** ∪ **B**):



N_B

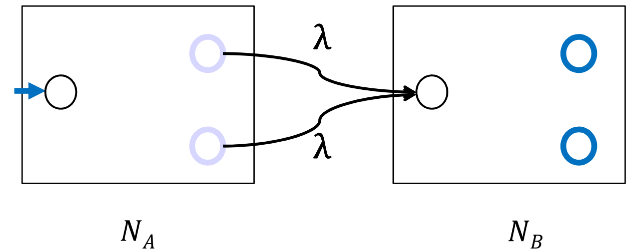
inductive step

Case (AB):



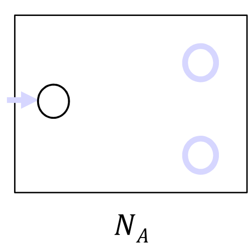
inductive step

Case (AB):



inductive step

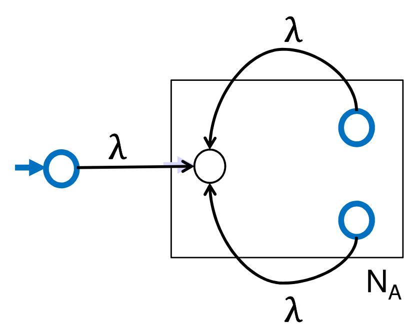
Case A*



□

inductive step

Case A*

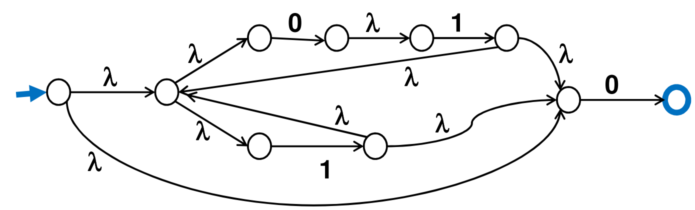


□

build an NFA for (01 ∪1)*0

solution

(01 ∪1)*0



NFAs and DFAs

Every DFA is an NFA

– DFAs have requirements that NFAs don't have

Can NFAs recognize more languages?

NFAs and DFAs

Every DFA is an NFA

– DFAs have requirements that NFAs don't have

Can NFAs recognize more languages? No!

Theorem: For every NFA there is a DFA that recognizes exactly the same language

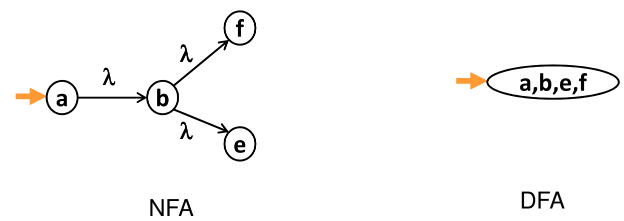
conversion of NFAs to a DFAs

- Proof Idea:
  - The DFA keeps track of ALL the states that the part of the input string read so far can reach in the NFA
  - There will be one state in the DFA for each *subset* of states of the NFA that can be reached by some string

conversion of NFAs to a DFAs

New start state for DFA

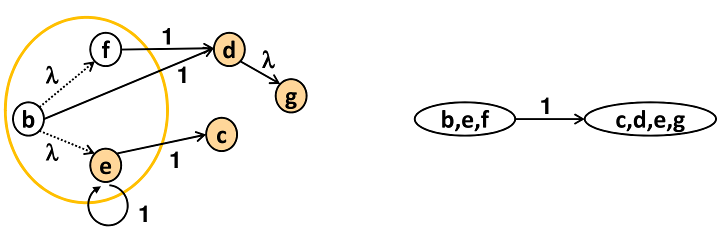
- The set of all states reachable from the start state of the NFA using only edges labeled λ



conversion of NFAs to a DFAs

For each state of the DFA corresponding to a set S of states of the NFA and each symbol s

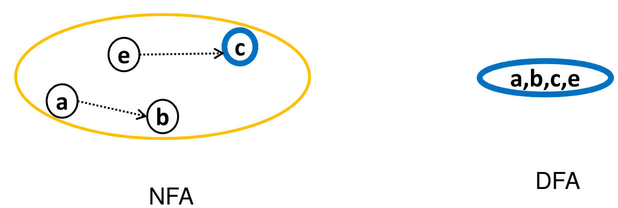
- Add an edge labeled **s** to state corresponding to T, the set of states of the NFA reached by
  starting from some state in S, then
  following one edge labeled by **s**, and
  then following some number of edges labeled by λ
- T will be $\varnothing$ if no edges from S labeled **s** exist



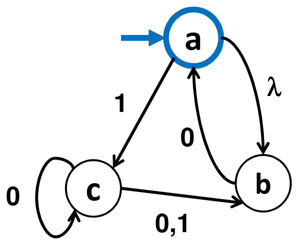
conversion of NFAs to a DFAs

Final states for the DFA

- All states whose set contain some final state of the NFA



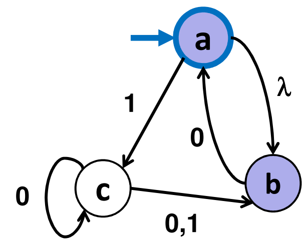
example: NFA to DFA



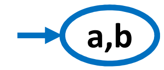
NFA

DFA

example: NFA to DFA

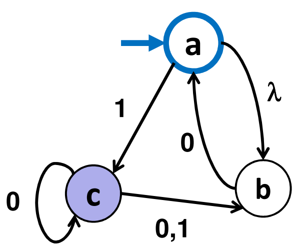


NFA

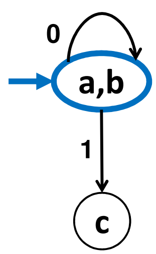


DFA

example: NFA to DFA

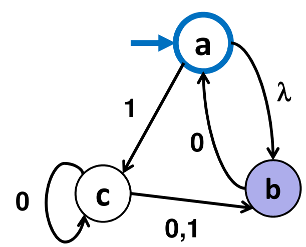


NFA

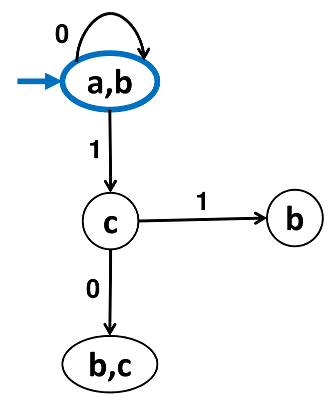


DFA

example: NFA to DFA

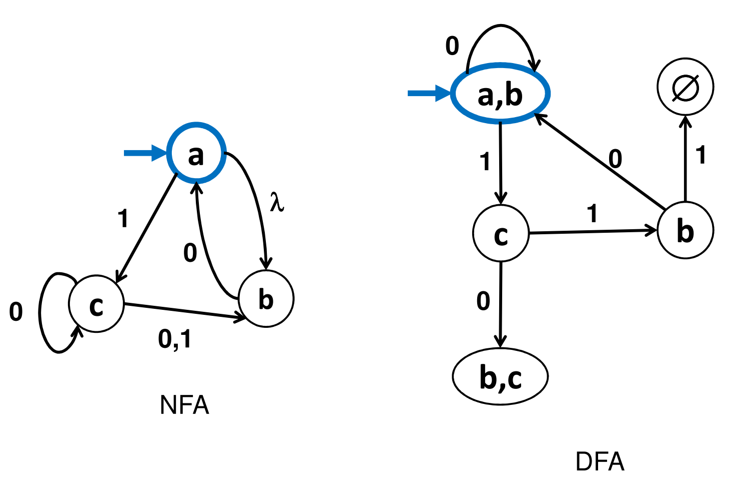


NFA

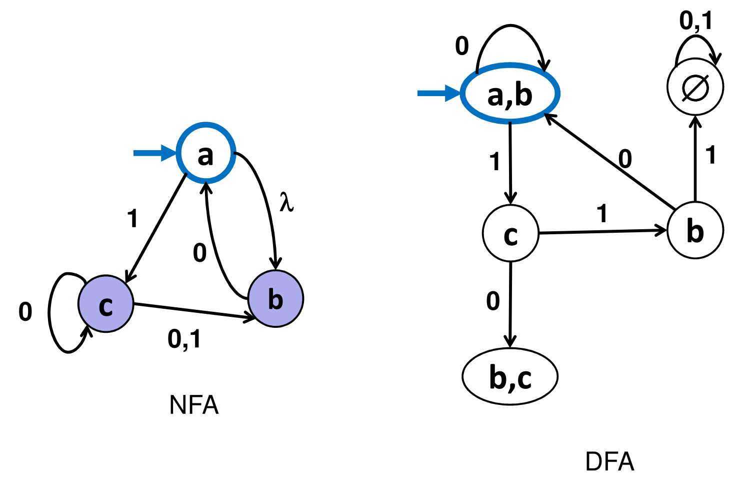


DFA

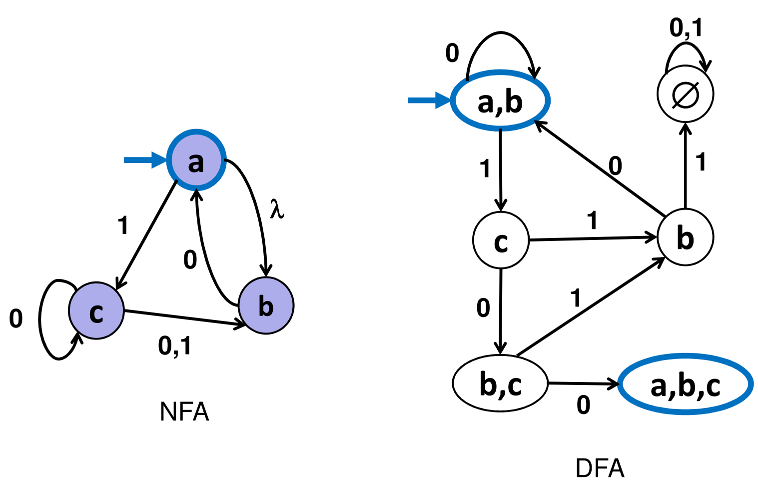
example: NFA to DFA



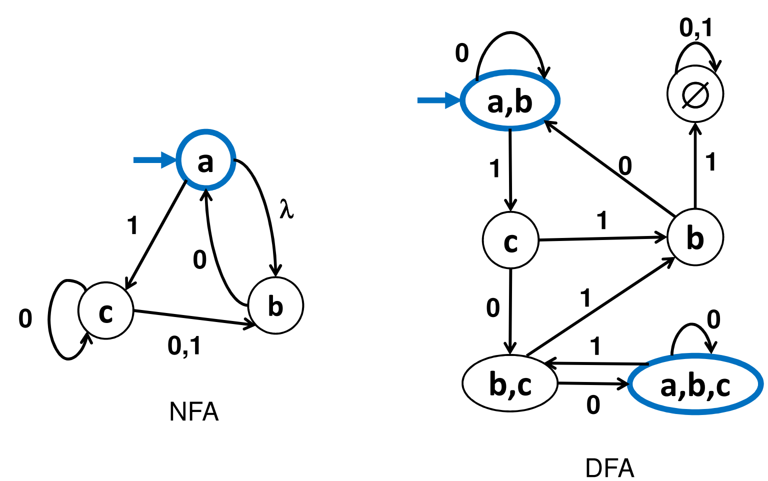
example: NFA to DFA



example: NFA to DFA



example: NFA to DFA



exponential blow-up in simulating nondeterminism

- In general the DFA might need a state for every subset of states of the NFA
  - Power set of the set of states of the NFA
  - n-state NFA yields DFA with at most 2^n states
  - We saw an example where roughly 2^n is necessary

    Is the n^{th} char from the end a 1?

- The famous "P=NP?" question asks whether a similar blow-up is always necessary to get rid of nondeterminism for polynomial-time algorithms