CSE 311: Foundations of Computing

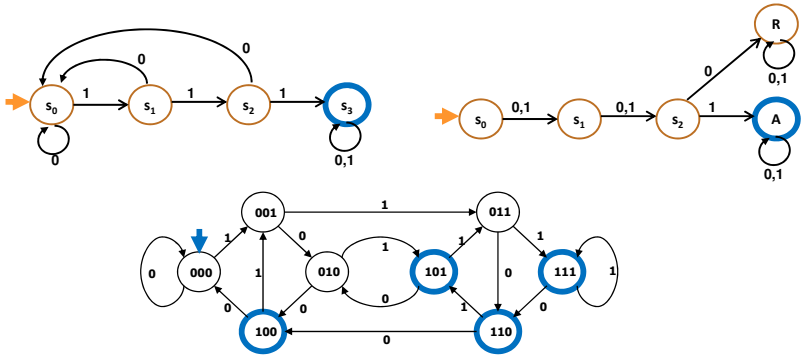
Fall 2013

Lecture 23: Finite state machines and minimization

FSM highlights

Finite state machines

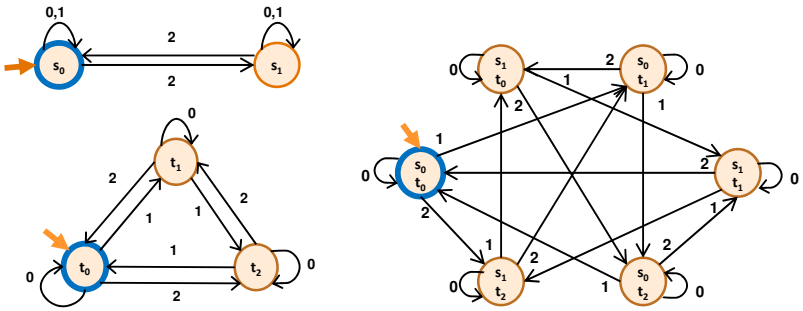
- States, transitions, start state, final states
- Languages recognized by FSMs



FSM highlights

Combining FSMs to check two properties at once

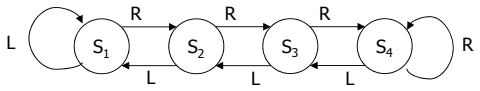
New states record states of both FSMs



state machines with output

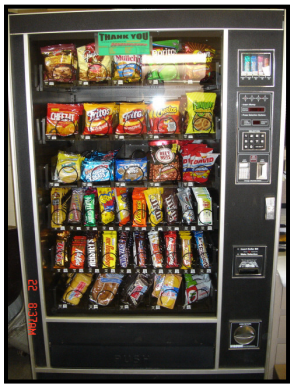
	Input		Output
State	L	R	
S_1	S_1	S_2	Beep
S_2	S_1	S_3	
S_3	S_2	S_4	
S_4	S_3	S_4	Beep

"Tug-of-war"

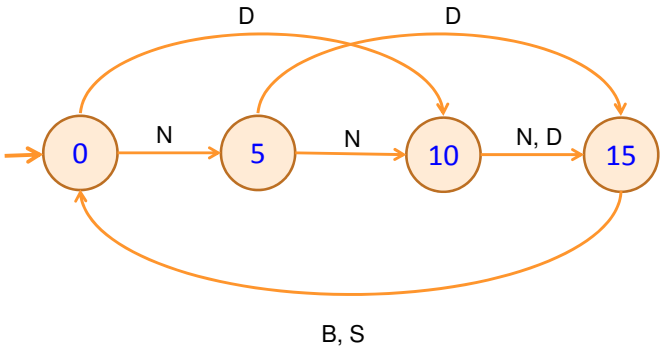


vending machine

Enter 15 cents in dimes or nickels
Press S or B for a candy bar

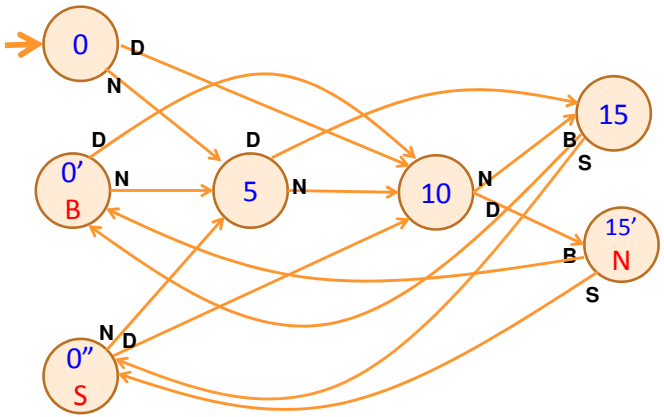


vending machine, v0.1



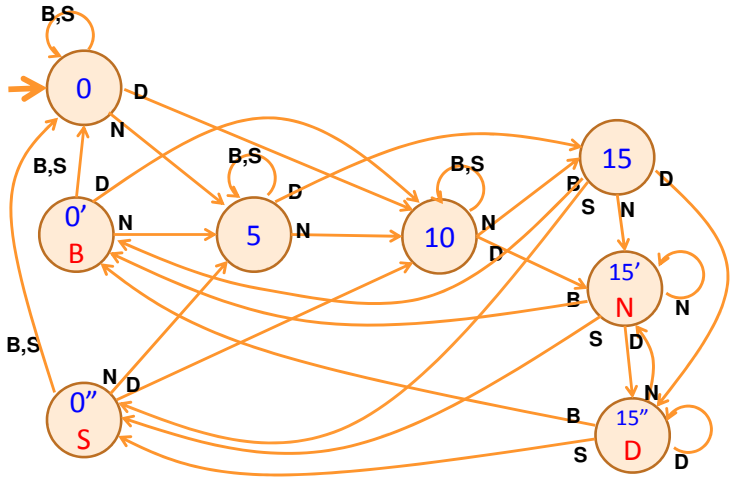
Basic transitions on N (nickel), D (dime), B (butterfinger), S (snickers)

vending machine, v0.2



Adding output to states: N – Nickel, S – Snickers, B – Butterfinger

vending machine, v1.0



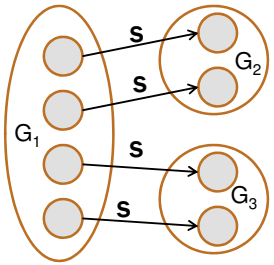
Adding additional "unexpected" transitions

state minimization

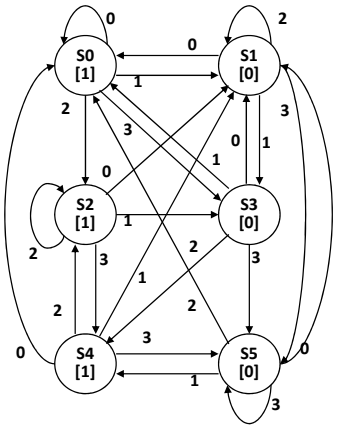
- Many different FSMs (DFAs) for the same problem
- Take a given FSM and try to reduce its state set by combining states
  - Algorithm will always produce the unique minimal equivalent machine (up to renaming of states) but we won't prove this

state minimization algorithm

1. Put states into groups based on their outputs (or whether they are final states or not)
2. Repeat the following until no change happens
   a. If there is a symbol ***s*** so that not all states in a group G agree on which group ***s*** leads to, split G into smaller groups based on which group the states go to on ***s***



state minimization example

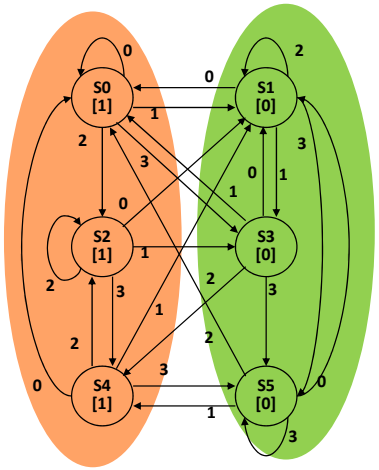


present state	next state 0	1	2	3	output
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

Put states into groups based on their outputs (or whether they are final states or not)

state minimization example

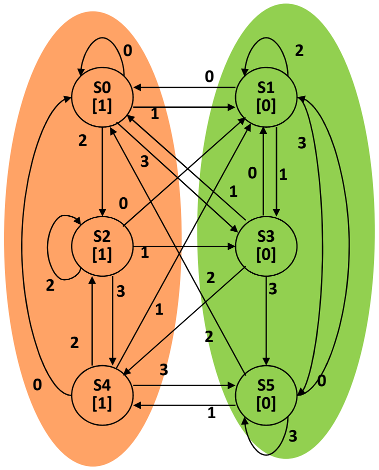


present state	next state 0	1	2	3	output
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

Put states into groups based on their outputs (or whether they are final states or not)

state minimization example



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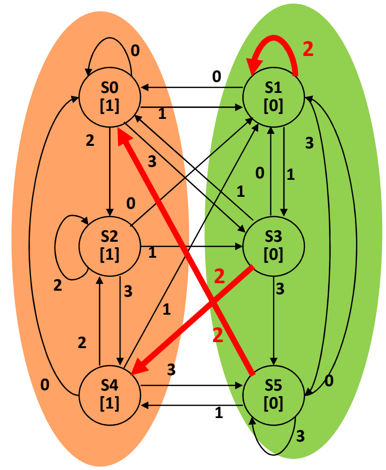
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

Put states into groups based on their outputs (or whether they are final states or not)

If there is a symbol ***s*** so that not all states in a group G agree on which group ***s*** leads to, split G based on which group the states go to on ***s***

state minimization example



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S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

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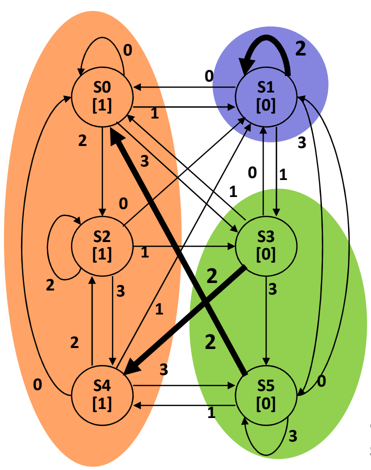
state minimization example



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S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

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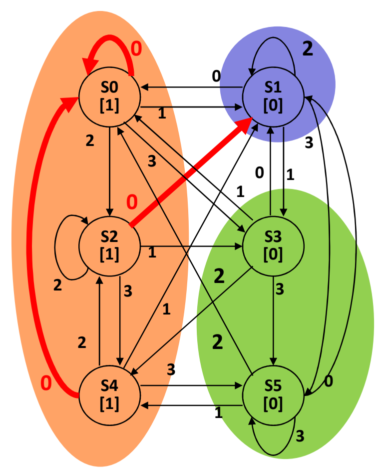
state minimization example



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S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

Put states into groups based on their outputs (or whether they are final states or not)

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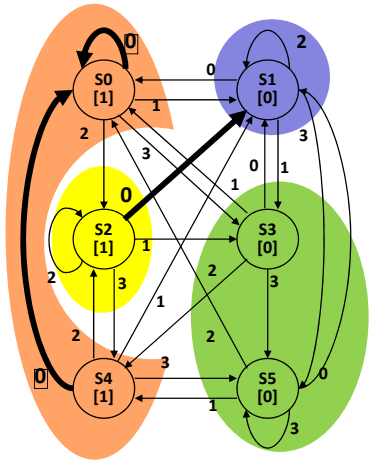
state minimization example



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S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

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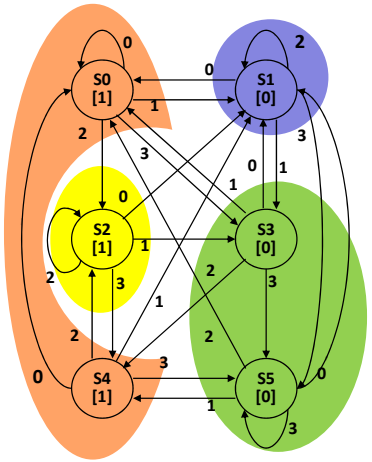
state minimization example



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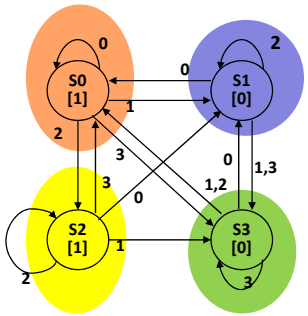
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

Can combine states S0-S4 and S3-S5.

In table replace all S4 with S0 and all S5 with S3

minimized machine



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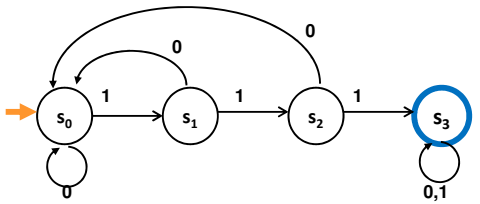
S1	S0	S3	S1	S3	0
S2	S1	S3	S2	S0	1
S3	S1	S0	S0	S3	0

state transition table

another way to look at DFAs

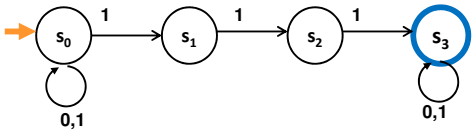
Definition: The label of a path in a DFA is the concatenation of all the labels on its edges in order

Lemma: x is in the language recognized by a DFA iff x labels a path from the start state to some final state



nondeterministic finite automaton (NFA)

- Graph with start state, final states, edges labeled by symbols (like DFA) but
  - Not required to have exactly 1 edge out of each state labeled by each symbol— can have 0 or >1
  - Also can have edges labeled by empty string λ
- **Definition:** x is in the language recognized by an NFA if and only if x labels a path from the start state to some final state



goal: NFA to recognize…

binary strings with even # of 1's that contain the substring 111