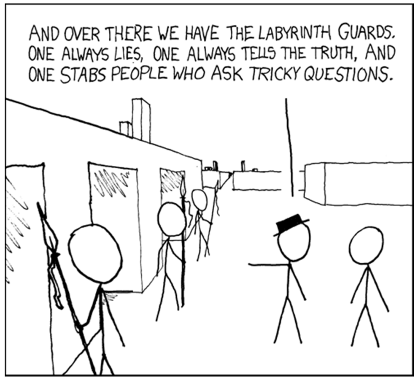
CSE 311: Foundations of Computing

Fall 2013

Lecture 2: Propositional logic



administrative

Course web: http://www.cs.washington.edu/311

Office hours: 10 hours available; check web

Homework #1: Posted. Turn in (stapled) at the start of class on Wednesday (Oct 2)

Extra credit: Not required to get a 4.0.
Counts separately.
In total, may raise grade by ~0.1

recall: connectives

p	$\neg p$
T	F
F	T

NOT

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

AND

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

XOR

recall: $p \rightarrow q$

Implication:

- *p* implies *q*
- whenever *p* is true *q* must be true
- if *p* then *q*
- *q* if *p*
- *p* is sufficient for *q*
- *p* only if *q*

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

recall: $p \rightarrow q$

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Implication:

- – *p* implies *q*
- – whenever *p* is true *q* must be true
- – if *p* then *q*
- – *q* if *p*
- – *p* is sufficient for *q*
- – *p* only if *q*

"If you behave, then I'll buy you ice cream."

What if you don't behave?

recall: converse, contrapositive, inverse

- **Implication:** $p \rightarrow q$
- **Converse:** $q \rightarrow p$
- **Contrapositive:** $\neg q \rightarrow \neg p$
- **Inverse:** $\neg p \rightarrow \neg q$

Are these the same?

recall: Biconditional: $p \leftrightarrow q$

- ***p* iff *q***
- ***p* is equivalent to *q***
- ***p* implies *q* and *q* implies *p***

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

recall: English and logic

```
You cannot ride the roller coaster
if you are under 4 feet tall unless
you are older than 16 years old.
```

- *q*: you can ride the roller coaster
- *r*: you are under 4 feet tall
- *s*: you are older than 16

digital circuits

Computing with logic

- T corresponds to 1 or "high" voltage
- F corresponds to 0 or "low" voltage

Gates

- Take inputs and produce outputs (functions)
- Several kinds of gates
- Correspond to propositional connectives

  Only symmetric ones (order of inputs irrelevant)

gates

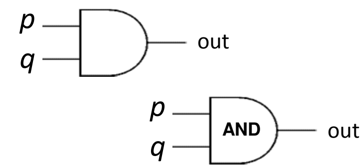
AND connective

$p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

AND gate

p	q	*out*
1	1	1
1	0	0
0	1	0
0	0	0



"block looks like D of AND"

gates

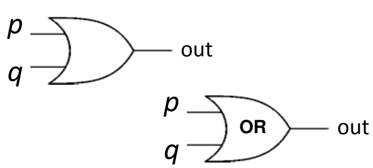
OR connective

$p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

OR gate

p	q	*out*
1	1	1
1	0	1
0	1	1
0	0	0



"arrowhead block looks like V"

gates

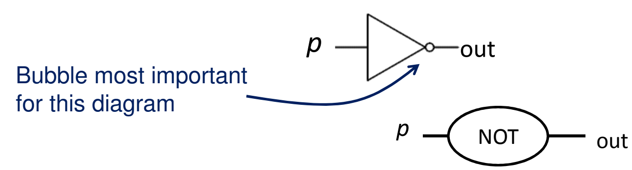
NOT connective

$\neg p$

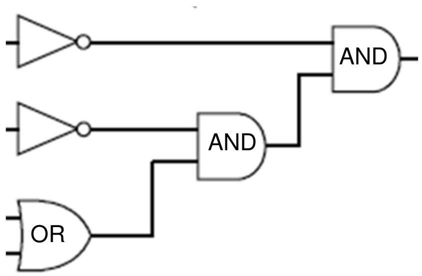
p	$\neg p$
T	F
F	T

NOT gate (inverter)

p	*out*
1	0
0	1

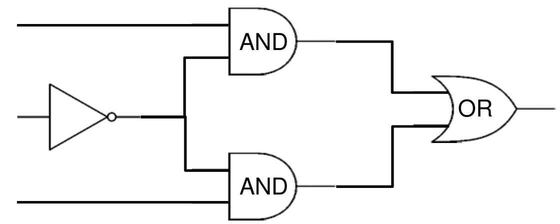


combinational logic circuits



Values get sent along wires connecting gates

combinational logic circuits



Wires can send one value to multiple gates

logical equivalence

Terminology: A compound proposition is a
- *Tautology* if it is always true
- *Contradiction* if it is always false
- *Contingency* if it can be either true or false

$p \vee \neg p$

$p \oplus p$

$(p \rightarrow q) \wedge p$

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

logical equivalence

p and q are *logically equivalent* if and only if $p \leftrightarrow q$ is a tautology
i.e. p and q have the same truth table

The notation $p \equiv q$ denotes p and q are logically equivalent

Example: $p \equiv \neg\neg p$

p	$\neg p$	$\neg\neg p$	$p \leftrightarrow \neg\neg p$

De Morgan's laws

$$\neg (p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg (p \vee q) \equiv \neg p \wedge \neg q$$

What are the negations of:

The Yankees and the Phillies will play in the World Series.

It will rain today or it will snow on New Year's Day.

De Morgan's laws

Example: $\neg (p \wedge q) \equiv (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$\neg (p \wedge q)$	$\neg (p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T						
T	F						
F	T						
F	F						

Law of Implication

$$(p \rightarrow q) \equiv (\neg p \vee q)$$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
T	T				
T	F				
F	T				
F	F				

computing equivalence

Describe an algorithm for computing if two logical expressions/circuits are equivalent.

What is the run time of the algorithm?

properties of logical connectives

- Identity
- Domination
- Idempotent
- Commutative
- Associative
- Distributive
- Absorption
- Negation

some equivalences related to implication

$p \rightarrow q$	$\equiv$	$\neg p \vee q$
$p \rightarrow q$	$\equiv$	$\neg q \rightarrow \neg p$
$p \vee q$	$\equiv$	$\neg p \rightarrow q$
$p \wedge q$	$\equiv$	$\neg (p \rightarrow \neg q)$
$p \leftrightarrow q$	$\equiv$	$(p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q$	$\equiv$	$\neg p \leftrightarrow \neg q$
$p \leftrightarrow q$	$\equiv$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg (p \leftrightarrow q)$	$\equiv$	$p \leftrightarrow \neg q$

going crazy yet? so did they...

George Boole
(of Boolean logic)

understanding connectives

- Reflect basic rules of reasoning and logic
- Allow manipulation of logical formulas
  - Simplification
  - Testing for equivalence
- Applications
  - Query optimization
  - Search optimization and caching
  - Artificial Intelligence
  - Program verification