CSE 311: Foundations of Computing I
Assignment #5
October 23, 2013
Due: October 30, 2013

Reading assignment: Read Sections 4.1-4.4 of the 7th edition or 3.4-3.6 of the 6th edition.

Problems

1. Compute the GCD of 1529 and 14039 using the Euclidean algorithm. Show the intermediate values that are computed.

2. Use the Euclidean algorithm to solve the following problems:
    a) Find the multiplicative inverse of 4 modulo 11.
    b) Find the multiplicative inverse of 7 modulo 22.
    c) Find the multiplicative inverse of 7 modulo 30.

3. Find a multiple of 2013 that ends with the digits 9999. Explain how you found your answer, including your intermediate calculations.

4. How many zeros are at the end of 3000! ? Justify your answer without computing 3000!.

5. Use induction to prove that $n^3 + 5n$ is a multiple of 3 for every positive integer n.

6. Prove that for every real number $x \neq 1$ and positive integer n, it holds true that

$$1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

7. Let x be any fixed real number with $x > -1$. Prove that $(1 + x)^n \geq 1 + nx$ for every integer $n \geq 0$.

Extra credit #1: Show that if $2^k + 1$ is a prime number, then the binary representation of k has at most one 1 in it. For instance, $2^4 + 1$ is prime and $4 = (100)_2$.

Extra credit #2: Suppose that m, n, p are positive integers such that $\gcd(m, n) = \gcd(m, p) = \gcd(n, p) = 1$. Show that if $N \geq 3mnp$, then any $N \times N$ chessboard can be tiled with tiles of sizes $m \times m, n \times n$, and $p \times p$.