CSE 311: Foundations of Computing I
Assignment #4
October 16, 2013
Due: October 23, 2013

Reading assignment: Read Sections 2.1-2.3 of either edition and Sections 4.1-4.2 of the 7th Edition (or 3.4-3.5 of the 6th Edition). Also read the Grading Guidelines on the course homepage.

Problems

1. Prove or disprove: For all sets A, B, C, if $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

2. Prove that for all sets A, B, C such that $C \neq \emptyset$,
$$A \times C = B \times C \quad \text{if and only if} \quad A = B.$$

3. Prove that $A = B$ if and only if $\mathcal{P}(A) = \mathcal{P}(B)$.

4. Each of the following functions maps the non-negative integers $\mathbb{N}$ to the non-negative integers $\mathbb{N}$. For each of the functions below, indicate the following: (i) its range, (ii) whether the function is one-to-one, (iii) whether the function is onto. Briefly justify your answers.
   a) $f(n) = n/2$ if n is even and $f(n) = (3n + 1)/2$ if n is odd.
   b) $f(n) = 2^n$
   c) $f(n) = n + 1$ if n is even, and $f(n) = n - 1$ if n is odd.
   d) $f(n) = n^2 - n + 1$
   e) $f(n)$ = the smallest integer k such that $2^k \geq n + 1$.

5. Prove that if n is an integer then $n^2 \bmod 6$ is either 0, 1, 3, or 4.

6. Let a, b be integers and c, m be positive integers. Prove that if $ac \equiv bc \pmod{cm}$, then $a \equiv b \pmod{m}$.

7. Let a be an integer and b, m be positive integers and define $c = a \bmod m$.
   Prove that $\{d: d \equiv a \pmod{bm}\} \subseteq \{d: d \equiv c \pmod{m}\}$.

Extra credit: Show that a positive integer is divisible by 11 if and only if the difference between the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.