CSE 311: Foundations of Computing I
Assignment #3
October 9, 2013
Due: October 16, 2013

Reading assignment: Read Sections 1.6-1.7 of the 7th edition (1.5-1.7 of 6th edition).   Also read the Grading Guidelines on the course homepage.

Problems

1. In the multi-player game, WiffnStuff, players form ad-hoc teams to solve puzzles and also have one-on-one duels of wits. Players get credit for all the puzzles they solve with their teammates but may be on different teams at different times during the game. Define predicate $T(x, t)$ to say that player x was on team t, $B(x, y)$ to say that player x beat player y in a duel of wits and $S(t, p)$ to say that team t solved puzzle p. (Unlike sports teams, team names are different whenever they have different members.) Express each of the following statements in predicate logic using the above predicates, the $\forall$ and $\exists$ quantifiers, and the usual connectives.
    a) Shafi and Silvio were on a team together.
    b) Some player beat every other player in a duel of wits.
    c) No team solved the Goldbach puzzle.
    d) Johan solved all the puzzles.
    e) The Wiles team solved only one of the puzzles.
    f) Alan was beaten by precisely one other player.

2. Express the following predicate logic statements in English in a natural form (without using variable names if you can).   Also, for each of the statements say whether it is true or false when the domain for each variable is (i) the integers (ii) the set of all rational numbers (iii) the set of all real numbers.
    a) $\forall x \forall y \left((x \neq y) \rightarrow ((x < y) \lor (y < x))\right)$
    b) $\forall x \forall y ((x < y) \rightarrow \exists z ((x < z) \land (z < y)))$
    c) $\forall x \forall y \forall z \left(((x < y) \land (y < z)) \rightarrow (x < z)\right)$

3. Using the logical inference rules from the lecture slides, as well as equivalences from the textbook, show that you can infer $r \rightarrow \neg q$ from $(p \lor r) \rightarrow (q \rightarrow s)$, t, and $(r \land t) \rightarrow \neg s$. How many rows would you need if you tried to do this using a truth table?

4. Using the logical inference rules from the lecture slides, as well as equivalences from the textbook, prove that given $\forall x (Q(x) \rightarrow \neg P(x))$, $\forall x ((P(x) \land \neg Q(x)) \rightarrow R(x))$, and $\exists x P(x)$ you can conclude that $\exists x R(x)$.

For questions 5-7 either formal inference using proof rules or English proofs are OK.

5. Prove that for every integer n, if $5n - 1$ is odd then n is even.   You should use the definitions of even and odd as given in class.   You can use the property shown in class that no number is both even and odd.   You can also use the fact that every number is even or odd.

6. Prove or disprove: For every integer $n > 1$, $n^2 + n - 1$ is prime.

7. Prove that for every non-zero rational number x, the number $1/x$ is also rational.

Extra credit: Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins.

On their ship, they decide to split the coins using the following scheme. The first pirate in alphabetical order becomes the chief pirate. The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it. If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain. Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent, what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.