CSE 311: Foundations of Computing I
Assignment #2
October 2, 2013
Due: October 9, 2013

Reading assignment: Read Sections 12.1-12.3 and 1.4 of 7th edition (11.1-11.3 and 1.3 of 6th edition). Also read the Grading Guidelines on the course homepage.

Problems

1. Use a sequence of basic logical equivalences (Table 6 in Sec. 1.3 of 7th or 1.2 of 6th) to prove that $\big(p \wedge (p \to q)\big) \to q$ is a tautology. Make sure to label each line with the equivalence you use.

2. A standard notation for the integer represented by a sequence of bits is to surround the bits by $(\cdots)_2$. For example, $(11)_2 = 3$ and $(0101)_2 = 5$. In this question, you will construct a circuit that takes a pair of two-bit integers $(x_1x_0)_2$ and $(y_1y_0)_2$ and computes the four output bits for their integer product $(z_3z_2z_1z_0)_2$.

   (a) Give sum-of-products forms for the two output bits of the product $(a_1a_0)_2$ of $(y_1y_0)_2$ and $(x_0)_2$. Do the same for the product of $(y_1y_0)_2$ and $(x_1)_2$ yielding $(b_1b_0)_2$. These are the bits produced as part of applying the usual elementary school method for multiplying numbers.

   (b) Use the minimized sum-of-products forms for one-bit adders given in class, together with the results of the above two products to produce sum-of-products forms for the output bits z_3, z_2, z_1, z_0. Some of the inputs you give to the one-bit adders may be constants. Use Boolean algebra to minimize the resulting sum-of-products form as a sum-of-products using only x_1, x_0, y_1, y_0.

   (c) Draw circuit diagrams for the results.

3. Translate these statements into English where Z(x) is "x is a Zombie" and E(x) is "eats brains" and A(x) is "afraid of the sun," and the domain consists of all people.
   a) $\exists x\, \big(Z(x) \wedge \neg E(x)\big)$
   b) $\forall x\, (\neg E(x) \to A(x))$
   c) $\exists x\, (Z(x) \to \big(E(x) \wedge \neg A(x)\big))$
   d) $\forall x\, (\big(E(x) \wedge A(x)\big) \to Z(x))$

4. Consider the following predicates:

    D(x) = “x likes to dance”
    B(x) = “x is a baby”
    M(x) = “is an expert in mixed martial arts” and
    E(x) = “eats brains.”

    Express each of these statements using quantifiers, logical connectives, and the above predicates. The domain consists of all people.

    (a) All babies like to dance and eat brains.
    (b) Nobody who likes to dance is an expert in mixed martial arts.
    (c) There is a mixed martial arts expert who dances unless he is a baby.
    (d) People who eat brains do not like dancing.

    Suppose that (a), (b), (c), and (d) are true. Is the following statement necessarily true: “There is a baby who is an expert in mixed martial arts” ?

5. Rewrite each of these statements so that the negations appear only next to predicate symbols (so that no negation is outside a quantifier or an expression involving logical connectives).

    a) $\neg\forall y\, \exists x\, P(x, y)$
    b) $\neg\exists x\, \left(Q(x) \land \forall y\, \neg P(x, y)\right)$
    c) $\neg\forall x\, \left(\forall y\, \neg\left(P(x) \rightarrow Q(x, y)\right)\right)$

6. Give examples of predicates and domains such that the statements $\exists x\, \left(P(x) \rightarrow Q(x)\right)$ and $\exists x\, P(x) \rightarrow \exists x\, Q(x)$ are not equivalent. Also give an example of predicates and domains where they **are** equivalent.

Extra credit: Design a boolean circuit that has six inputs, and one output where the output is 1 if exactly three of the inputs are 1, and is 0 otherwise. Your circuit should use as few gates as possible. Provide a brief explanation as to how your circuit works.

As a starting point, the brute force approach, which looks at every possible way of having exactly three true inputs takes 1 OR gate, 20 AND gates, and 6 NOT gates. We don't know what the best possible result is, One instructor came up with a circuit that uses 13 AND gates, 3 XOR gates, 1 OR gate, and 6 NOT Gates.