CSE311: Worksheet, May 10, 2012

1. Example of a subtle error in a proof by induction:

   "All horses are the same color."

   You can find a pseudo-proof and an explanation in the wikipedia web page:
   `http://en.wikipedia.org/wiki/All_horses_are_the_same_color`

2. "Define the Fibonnaci numbers as follows: $f(0) = 0, f(1) = 1$, and $f(n) = f(n-2) + f(n-1)$ for all integers $n > 1$. Prove by induction that, for all nonnegative integers n, the number of iterations used by Euclid's algorithm to compute $gcd(f(n+1), f(n))$ is n."

   Proof: The basis is $n = 0$, and indeed $gcd(1,0)$ uses no iterations. For the induction step, the first iteration changes the arguments from $(f(n+1), f(n))$ to $(f(n), f(n-1))$, and the induction hypothesis says it takes $n-1$ more iterations to finish the computation.

   The only hitch is that the theorem is false for almost all values of n. For your entertainment, find the flaw in the proof. (It's not hard to find once you know it's false, but I find the proof absolutely convincing if you don't suspect it's false.)

3. Definition of a full binary tree:

   (a) BASIS: There a binary tree with a single vertex (That vertex is also the root of the tree).

   (b) RECURRENCE: Two disjoint full binary trees T_1 and T_2 can form a full binary tree. Create a new vertex as the root. Use two edges to join that root with the roots of T_1 and T_2.

   Prove that every full binary tree with k leaves has $k-1$ internal vertices.

4. Prove the following:

   $$1 + \frac{1}{2^2} + \frac{1}{3^2} + ... + \frac{1}{n^2} \leq 2 \ , \ n \geq 1$$

   Hint1: Try replacing the right hand side of the inequality with something that will make the statement stronger.

   Hint2: Ask the TA.

5. Let L denote a language using alphabet $\{0, 1\}$:

(a) BASIS: $\epsilon \in L$ (The empty string is in L).

(b) RECURRENCE: If $v, u \in L$ then both $0v1u$ and $1v0u$ are also in L.

Prove that L is characterized as the collection of binary strings with an equal number of 0's and 1's. The definition of language is to be given in class.