CSE311 Quiz Section: April 05, 2012

1. Prove that $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$ by rewriting with equivalences.

$$\begin{aligned}
(p \to r) \wedge (q \to r) &\equiv (p \vee q) \to r && \\
&\equiv \neg(p \vee q) \vee r && \text{Law of implication} \\
&\equiv (\neg p \wedge \neg q) \vee r && \text{DeMorgan's} \\
&\equiv (\neg p \vee r) \wedge (\neg q \vee r) && \text{Distributive} \\
&\equiv (p \to r) \wedge (q \to r) && \text{Law of implication}
\end{aligned}$$

2. Prove that $(p \wedge q) \to (p \to q)$ is a tautology by rewriting with equivalences.

$$\begin{aligned}
(p \wedge q) \to (p \to q) &\equiv T && \\
\neg(p \wedge q) \vee (p \to q) & && \text{Law of implication} \\
\neg(p \wedge q) \vee (\neg p \vee q) & && \text{Law of implication} \\
(\neg p \vee \neg q) \vee (\neg p \vee q) & && \text{DeMorgan} \\
\neg p \vee \neg q \vee \neg p \vee q & && \text{Associative} \\
\neg p \vee \neg q \vee q \vee \neg p & && \text{Commutative} \\
\neg p \vee T \vee \neg p & && \text{Negation} \\
\neg p \vee T & && \text{Domination} \\
T & && \text{Domination}
\end{aligned}$$

3. Assume you have a table with $n + 1$ columns. The first n correspond to the propositions (or variables of the function f) and the last one to the output of f. There are 2^n possible different assignments to n variables. For each such assignment we have a different row for the output of the function. Notice that f is defined by exactly by the output values of this last column. If the output of two functions differs in at least one row then the functions are different. Therefore by counting all possible different assignments to that last column we can see how many different functions exist. Imagine we have k cells in that column. If we try to assign a value to the first cell, we have two options (T or F). For the second cell we still have two options, but depending on the value of the first we get a different function. In total we have 4 possible assignments for the first two cells. Generally for k cells we have 2^k different assignments (much like the number of different assignments for the variables). Since in our case the column is of size $k = 2^n$, we get 2^{2^n} different functions.

The rest of the exercises are solved in the book.