CSE 311: Foundations of Computing I Assignment #6 May 4, 2012 due: Monday, May 14, 1:30 p.m.

Textbook numbering labeled "6th edition" refers to the textbook's Sixth Edition. Numbering that is unlabeled refers to the Seventh Edition.

- 1. Definition 4 in Section 5.3 [6th edition: Definition 5 in Section 4.3] defines an extended binary tree. If  $T_1$  and  $T_2$  are both empty,  $T_1 \cdot T_2$  is called a *leaf*. For example, the last tree in Figure 3 of Section 5.3 [6th edition: Section 4.3] has 2 leaves and the next to last tree in that figure has 3 leaves. The *height* of an extended binary tree is the distance from the root to the farthest leaf. All the trees in Step 3 of Figure 3 have height 2. (Note that the height is considered to be 2 rather than 3: it's the number of edges on the longest root-to-leaf path rather than the number of nodes.) By induction, prove that for any positive integer n, any extended binary tree with n leaves has height at least  $\log_2 n$ . Be careful of the possibility that your tree has one empty subtree and one nonempty subtree. (Hint: it will be simplest if your induction mirrors the recursive definition given in Definition 4 [6th edition: Definition 5].)
- 2. Section 9.5 [6th edition: Section 8.5], exercise 16.
- 3. Section 9.5 [6th edition: Section 8.5], exercise 40. The answer to part (b) should show that there is a one-to-one correspondence between the equivalence classes of R and the elements of a very familiar mathematical set.
- 4. Page 635 [6th edition: Page 584], Supplementary Exercise 10. ("A relation R is called circular ...")