CSE 311 Foundations of Computing I

Lecture 25

Finite State Machines: Output and Minimization

Autumn 2012

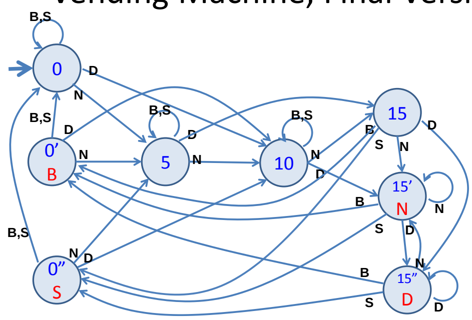
Announcements

- Reading assignments
  - 7^{th} Edition, Sections 13.3 and 13.4
  - 6^{th} Edition, Section 12.3 and 12.4
  - 5^{th} Edition, Section 11.3 and 11.4

State Machines with Output Vending Machine

Enter 15 cents in dimes or nickels
Press S or B for a candy bar

Vending Machine, Final Version

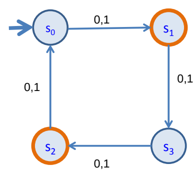


State Minimization

- Many different FSMs (DFAs) for the same problem
- Take a given FSM and try to reduce its state set by combining states
  - Algorithm will always produce the unique minimal equivalent machine (up to renaming of states) but we won't prove this

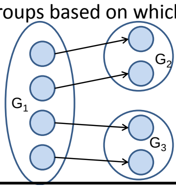
State minimization

- Recognizing strings with odd length

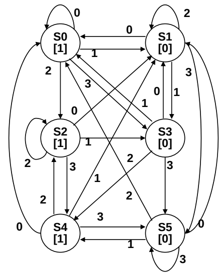


State minimization algorithm

1. Put states into groups based on their outputs (or whether they are final states or not)
2. Repeat the following until no change happens
   a. If there is a symbol ***s*** so that not all states in a group G agree on which group ***s*** leads to, split G into smaller groups based on which group the states go to on ***s***



State Minimization Example

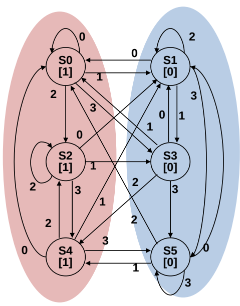


present state	next state 0	1	2	3	output
S0	S0	S1	S2	S3	1
S1	S0	S3	S1	S5	0
S2	S1	S3	S2	S4	1
S3	S1	S0	S4	S5	0
S4	S0	S1	S2	S5	1
S5	S1	S4	S0	S5	0

state transition table

Put states into groups based on their outputs (or whether they are final states or not)

State Minimization Example



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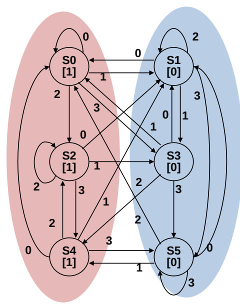
state transition table

Put states into groups based on their outputs (or whether they are final states or not)

State Minimization Example



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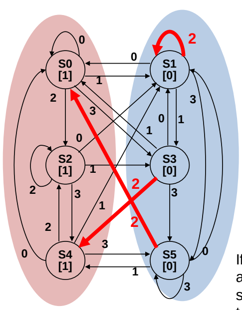
state transition table

Put states into groups based on their outputs (or whether they are final states or not)

If there is a symbol ***s*** so that not all states in a group G agree on which group ***s*** leads to, split G based on which group the states go to on ***s***

State Minimization Example



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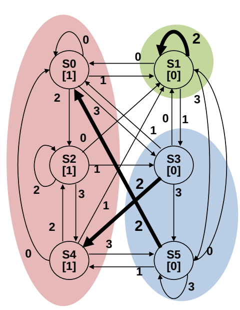

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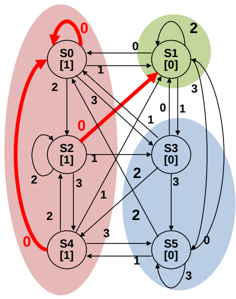

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state transition table

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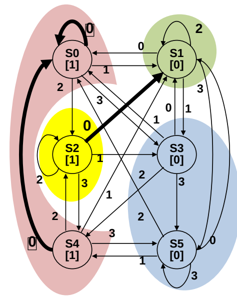
State Minimization Example



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state transition table

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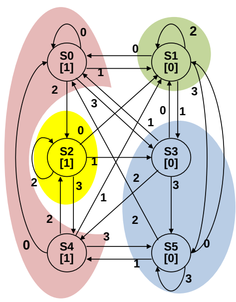

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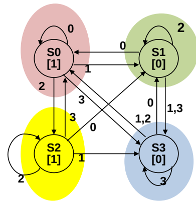
S5	S1	S4	S0	S5	0

state transition table

Can combine states S0-S4 and S3-S5.

In table replace all S4 with S0 and all S5 with S3

Minimized Machine



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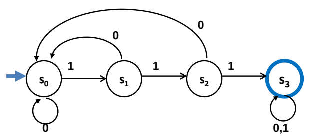
S2	S1	S3	S2	S0	1
S3	S1	S0	S0	S3	0

state transition table

Another way to look at DFAs

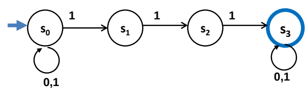
Definition: The label of a path in a DFA is the concatenation of all the labels on its edges in order

Lemma: x is in the language recognized by a DFA iff x labels a path from the start state to some final state



Nondeterministic Finite Automaton (NFA)

- Graph with start state, final states, edges labeled by symbols (like DFA) but
  - Not required to have exactly 1 edge out of each state labeled by each symbol - can have 0 or >1
  - Also can have edges labeled by empty string λ
- Definition: x is in the language recognized by an NFA iff x labels a path from the start state to some final state



Design an NFA to recognize the set of binary strings that contain 111 or have an even # of 1's

Finite state machines and regular expressions

- Every regular expression can be recognized by a NFA
- Every NSA can be converted into an equivalent regular expression
- Every NFA can be converted into an equivalent DFA

1 and 3 will be sketched in class

NFAs and DFAs

Every DFA **is** an NFA

- DFAs have requirements that NFAs don't have

Can NFAs recognize more languages? No!

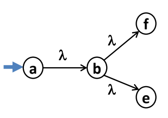
Theorem: For every NFA there is a DFA that recognizes exactly the same language

Conversion of NFAs to a DFAs

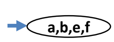
- Proof Idea:
  - The DFA keeps track of ALL the states that the part of the input string read so far can reach in the NFA
  - There will be one state in the DFA for each *subset* of states of the NFA that can be reached by some string

Conversion of NFAs to a DFAs

- New start state for DFA
  - The set of all states reachable from the start state of the NFA using only edges labeled λ



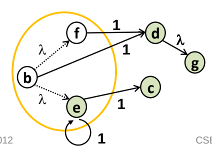
NFA

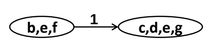


DFA

Conversion of NFAs to a DFAs

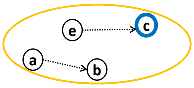
- For each state of the DFA corresponding to a set S of states of the NFA and each symbol **s**
  - Add an edge labeled **s** to state corresponding to T, the set of states of the NFA reached by
    - starting from some state in S, then
    - following one edge labeled by **s**, and
    - then following some number of edges labeled by λ
  - T will be ∅ if no edges from S labeled **s** exist





Conversion of NFAs to a DFAs

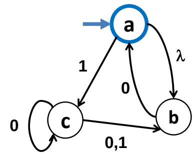
- Final states for the DFA
  - All states whose set contain some final state of the NFA



NFA

DFA

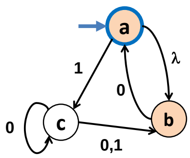
Example: NFA to DFA

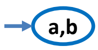


NFA

DFA

Example: NFA to DFA

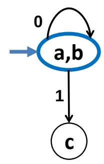


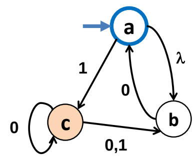


NFA

DFA

Example: NFA to DFA

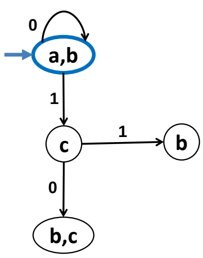


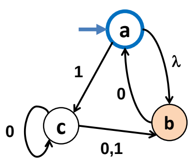


NFA

DFA

Example: NFA to DFA

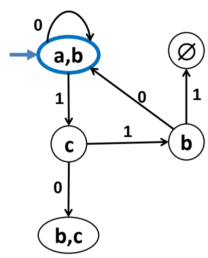


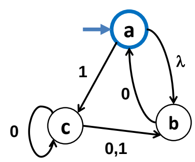


NFA

DFA

Example: NFA to DFA





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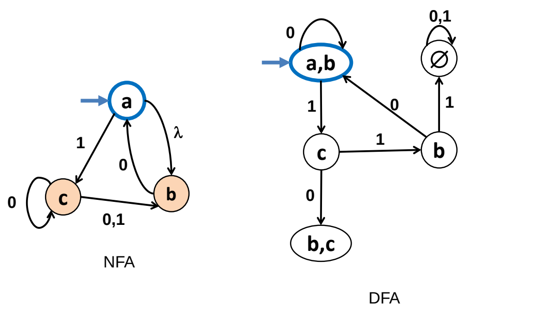

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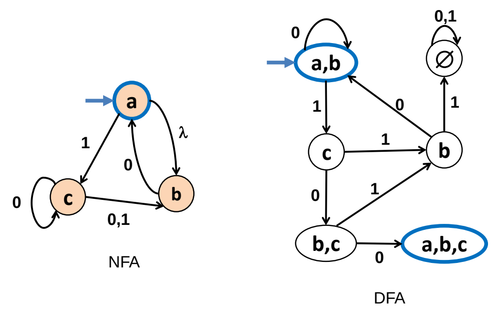
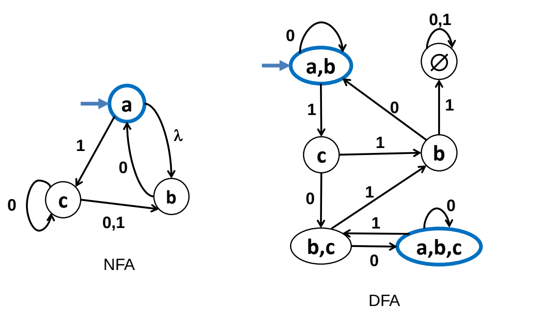


Example: NFA to DFA



Exponential blow-up in simulating nondeterminism

- In general the DFA might need a state for every subset of states of the NFA
  - Power set of the set of states of the NFA
  - n-state NFA yields DFA with at most 2^n states
  - We saw an example where roughly 2^n is necessary
    - Is the 10^{th} char from the end a 1?
- The famous "P=NP?" question asks whether a similar blow-up is always necessary to get rid of nondeterminism for polynomial-time algorithms