

# CSE 311 Foundations of Computing I

Lecture 24  
Finite State Machines  
Autumn 2012

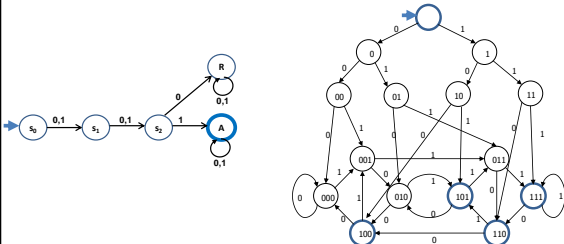
## Announcements

- Reading assignments
  - 7<sup>th</sup> Edition, Sections 13.3 and 13.4
  - 6<sup>th</sup> Edition, Section 12.3 and 12.4
  - 5<sup>th</sup> Edition, Section 11.3 and 11.4

## Last lecture highlights

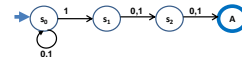
Finite state machines

- States, transitions, start state, final states
- Languages recognized by FSMs



## Non-determinism

- A non-deterministic finite automaton (NFA) allows multiple outputs to have the same label
- A string  $s$  is accepted by a NFA if there is some path through the NFA with labels  $s$  that reaches an accepting state

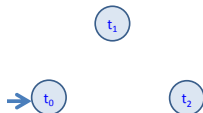


## Strings over $\{0, 1, 2\}^*$

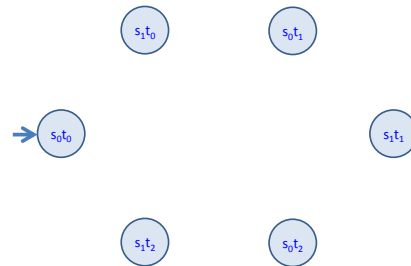
$M_1$ : Strings with an even number of 2's



$M_2$ : Strings where the sum of digits mod 3 is 0



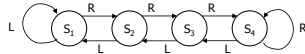
## Recognize strings with an even number of 2's and a mod 3 sum of 0



# State machines with output

State	Input		Output
	L	R	
$s_1$	$s_1$	$s_2$	Beep
$s_2$	$s_1$	$s_3$	
$s_3$	$s_2$	$s_4$	
$s_4$	$s_3$	$s_4$	Beep

"Tug-of-war"



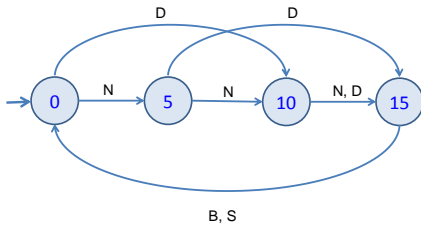
# Vending Machine



Enter 15 cents in dimes or nickels  
Press S or B for a candy bar

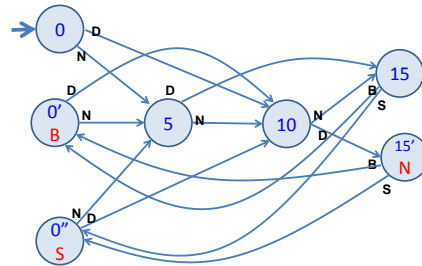


# Vending Machine, Version 1



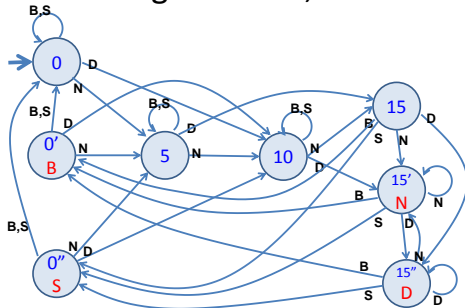
Basic transitions on N (nickel), D (dime), B (butterfinger), S (snickers)

# Vending Machine, Version 2



Adding output to states: N – Nickel, S – Snickers, B – Butterfinger

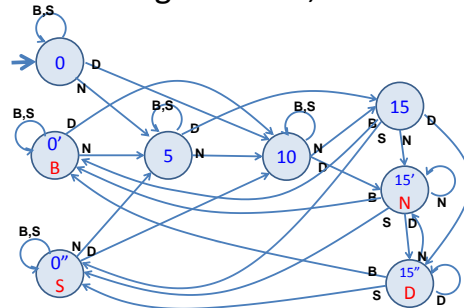
# Vending Machine, Final Version



Adding additional "unexpected" transitions

Slide from Nov 21, 2011 Lecture

# Vending Machine, Final Version

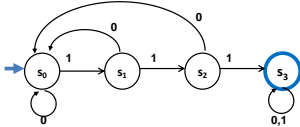


Find the bug!

## Another way to look at DFAs

Definition: The label of a path in a DFA is the concatenation of all the labels on its edges in order

Lemma:  $x$  is in the language recognized by a DFA iff  $x$  labels a path from the start state to some final state

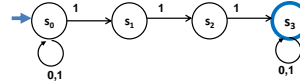


## Nondeterministic Finite Automaton (NFA)

- Graph with start state, final states, edges labeled by symbols (like DFA) but

- Not required to have exactly 1 edge out of each state labeled by each symbol - can have 0 or  $>1$
- Also can have edges labeled by empty string  $\lambda$

- Definition:  $x$  is in the language recognized by an NFA iff  $x$  labels a path from the start state to some final state



Design an NFA to recognize the set of binary strings that contain 111 or have an even # of 1's

## Three ways of thinking about NFAs

- Outside observer: Is there a path labeled by  $x$  from the start state to some final state?
- Perfect guesser: The NFA has input  $x$  and whenever there is a choice of what to do it magically guesses a good one (if one exists)
- Parallel exploration: The NFA computation runs all possible computations on  $x$  step-by-step at the same time in parallel