

CSE 311 Foundations of Computing I

Lecture 21
Relations
Autumn 2012

Autumn 2012

CSE 311

1

Announcements

- Reading assignments
 - 7th Edition, Section 9.1 and pp. 594-601
 - 6th Edition, Section 8.1 and pp. 541-548
 - 5th Edition, Section 7.1 and pp. 493-500
- Upcoming topics
 - Relations
 - Finite State Machines

Autumn 2012

CSE 311

2

Definition of Relations

Let A and B be sets,
A **binary relation from A to B** is a subset of $A \times B$

Let A be a set,
A **binary relation on A** is a subset of $A \times A$

Autumn 2012

CSE 311

3

Relation Examples

$$R_1 = \{(a, 1), (a, 2), (b, 1), (b, 3), (c, 3)\}$$

$$R_2 = \{(x, y) \mid x \equiv y \pmod{5}\}$$

$$R_3 = \{(c_1, c_2) \mid c_1 \text{ is a prerequisite of } c_2\}$$

$$R_4 = \{(s, c) \mid \text{student } s \text{ had taken course } c\}$$

Autumn 2012

CSE 311

4

Properties of Relations

Let R be a relation on A

R is **reflexive** iff $(a, a) \in R$ for every $a \in A$

R is **symmetric** iff $(a, b) \in R$ implies $(b, a) \in R$

R is **antisymmetric** iff $(a, b) \in R$ and $a \neq b$ implies $(b, a) \notin R$

R is **transitive** iff $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$

Autumn 2012

CSE 311

5

Combining Relations

Let R be a relation from A to B

Let S be a relation from B to C

The composite of R and S, $S \circ R$ is the relation from A to C defined

$$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}$$

Autumn 2012

CSE 311

6

Examples

$(a,b) \in \text{Parent}$: b is a parent of a

$(a,b) \in \text{Sister}$: b is a sister of a

What is Sister \circ Parent?

What is Parent \circ Sister?

Autumn 2012

$$S \circ R = \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}$$

Examples

Using the relations: Parent, Child, Brother, Sister, Sibling, Father, Mother, Husband, Wife express -

Uncle: b is an uncle of a

Cousin: b is a cousin of a

Autumn 2012

CSE 311

8

Powers of a Relation

$$R^2 = R \circ R = \{(a, c) \mid \exists b \text{ such that } (a, b) \in R \text{ and } (b, c) \in R\}$$

$$R^0 = \{(a, a) \mid a \in A\}$$

$$R^1 = R$$

$$R^{n+1} = R^n \circ R$$

Autumn 2012

CSE 311

9

How is  related to  ?



Anderson



Copernicus

Autumn 2012

CSE 311

10

How is  related to  ?

Suciu

Copernicus

Autumn 2012

CSE 311

11

<http://genealogy.math.ndsu.nodak.edu/>

Mathematics Genealogy Project

Edward Delano Lazowska

MathSciNet

Ph.D.: University of Toronto 1977 

Dissertation: Characterizing Service Time and Response Time Distributions in Queueing Network Models of Computer Systems

Advisor: Kenneth Clem Sevcik

Students:

Click [here](#) to see the students listed in chronological order.

service of the NDSU Department of Mathematics in association with the American Mathematical Society

Please email us with feedback.

Name	School	Year	Descendants
Thomas Anderson	University of Washington	1991	54
Robert Bedichek	University of Washington	1994	
John Bennett	University of Washington	1988	9
Brian Berndt	University of Washington	1990	16
Jeffrey Chase	University of Washington	1995	7
Sung Chung	University of Washington	1990	
Edward Felten	University of Washington	1993	8
Richard Garner	University of Washington	1982	
Patricia Jacobson	University of Washington	1984	
Henry (Hank) Levy	University of Washington	1981	123
Yi-Bing Lin	University of Washington	1990	13

12



Nicolaus Copernicus	Nicolaus Copernicus
Georg Rheticus	Georg Rheticus
Moritz Steinmetz	Moritz Steinmetz
Christoph Meurer	Christoph Meurer
Philipp Muller	Philipp Muller
Erhard Weigel	Erhard Weigel
Gottfried Leibniz	Gottfried Leibniz
Noclas Malebranache	Noclas Malebranache
Jacob Bernoulli	Jacob Bernoulli
Johann Bernoulli	Johann Bernoulli
Leonhard Euler	Leonhard Euler
Joseph Lagrange	Joseph Lagrange
Jean-Baptiste Fourier	Michel Chasles
Gustav Dirichlet	H. A. (Hubert Anson) Newton
Rudolf Lipschitz	E. H. (Eliakim Hastings) Moore
Felix Klein	Oswald Veblen
C. L. Ferdinand Lindemann	Alonzo Church
Herman Minkowski	Hartley Rogers, Jr.
Constantin Caratheodory	Patrick Carl Fischer
Georg Aumann	Albert Ronald da Silva Meyer
Friedrich Bauer	Val Tannen
Manfred Paul	Dan Suciuc
Ernst Mayr	
Richard Anderson	

Autumn 2012 CSE 311 15

n-ary relations

Let A_1, A_2, \dots, A_n be sets. An n-ary relation on these sets is a subset of $A_1 \times A_2 \times \dots \times A_n$.

Autumn 2012 CSE 311 16

Relational databases

STUDENT			
Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Von Neuman	481080220	555	3.78
Russell	238082388	022	3.85
Einstein	238001920	022	2.11
Newton	1727017	333	3.61
Karp	348882811	022	3.98
Bernoulli	2921938	022	3.21

Autumn 2012 CSE 311 17

Relational databases

STUDENT	Student_Name	ID_Number	Office	GPA	Course
Knuth	328012098	022	4.00	CSE311	
Knuth	328012098	022	4.00	CSE351	
Von Neuman	481080220	555	3.78	CSE311	
Russell	238082388	022	3.85	CSE312	
Russell	238082388	022	3.85	CSE344	
Russell	238082388	022	3.85	CSE351	
Newton	1727017	333	3.61	CSE312	
Karp	348882811	022	3.98	CSE311	
Karp	348882811	022	3.98	CSE312	
Karp	348882811	022	3.98	CSE344	
Karp	348882811	022	3.98	CSE351	
Bernoulli	2921938	022	3.21	CSE351	

What wrong?

Relational databases

STUDENT

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Von Neuman	481080220	555	3.78
Russell	238082388	022	3.85
Einstein	238001920	022	2.11
Newton	1727017	333	3.61
Karp	348882811	022	3.98
Bernoulli	2921938	022	3.21

Better

TAKES

ID_Number	Course
328012098	CSE311
328012098	CSE351
481080220	CSE311
238082388	CSE312
238082388	CSE344
238082388	CSE351
1727017	CSE312
348882811	CSE311
348882811	CSE312
348882811	CSE344
348882811	CSE351
2921938	CSE351

Database Operations: Projection

Find all offices: $\Pi_{\text{Office}}(\text{STUDENT})$

Office
022
555
333

Find offices and GPAs: $\Pi_{\text{Office}, \text{GPA}}(\text{STUDENT})$

Office	GPA
022	4.00
555	3.78
022	3.85
022	2.11
333	3.61
022	3.98
022	3.21

Autumn 2012

CSE 311

20

Database Operations: Selection

Find students with GPA > 3.9 : $\sigma_{\text{GPA}>3.9}(\text{STUDENT})$

Student_Name	ID_Number	Office	GPA
Knuth	328012098	022	4.00
Karp	348882811	022	3.98

Retrieve the name and GPA for students with GPA > 3.9 :
 $\Pi_{\text{Student_Name}, \text{GPA}}(\sigma_{\text{GPA}>3.9}(\text{STUDENT}))$

Student_Name	GPA
Knuth	4.00
Karp	3.98

Autumn 2012

CSE 311

21

Database Operations: Natural Join

Student \bowtie Takes

Student_Name	ID_Number	Office	GPA	Course
Knuth	328012098	022	4.00	CSE311
Knuth	328012098	022	4.00	CSE351
Von Neuman	481080220	555	3.78	CSE311
Russell	238082388	022	3.85	CSE312
Russell	238082388	022	3.85	CSE344
Russell	238082388	022	3.85	CSE351
Newton	1727017	333	3.61	CSE312
Karp	348882811	022	3.98	CSE311
Karp	348882811	022	3.98	CSE312
Karp	348882811	022	3.98	CSE344
Karp	348882811	022	3.98	CSE351
Bernoulli	2921938	022	3.21	CSE351