CSE 311 Foundations of Computing I

Lecture 19
Recursive Definitions: Context-Free Grammars and Languages
Autumn 2012

Announcements

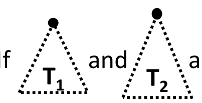
- Reading assignments
  - 7th Edition, pp. 878-880 and pp. 851-855
  - 6th Edition, pp. 817-819 and pp. 789-793
  - 5th Edition, pp. 766 and pp. 743-748
- For Friday, November 9
  - 7th Edition, Section 9.1 and pp. 594-601
  - 6th Edition, Section 8.1 and pp. 541-548
  - 5th Edition, Section 7.1 and pp. 493-500

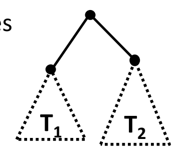
Highlight from last lecture: Structural Induction

How to prove ∀x∈S. P(x) is true:

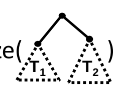
- **Base Case:** Show that P is true for all specific elements of S mentioned in the *Basis step*
- **Inductive Hypothesis:** Assume that P is true for some arbitrary values of each of the existing named elements mentioned in the *Recursive step*
- **Inductive Step:** Prove that P holds for each of the new elements constructed in the *Recursive step* using the named elements mentioned in the Inductive Hypothesis
- Conclude that ∀x∈S. P(x)

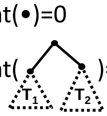
Rooted Binary trees

- Basis: • is a rooted binary tree
- Recursive Step: If  are rooted binary trees then so is:



Functions defined on rooted binary trees

- size(•)=1
- size() = 1+size(T_1)+size(T_2)

- height(•)=0
- height()=1+max{height(T_1),height(T_2)}

For every rooted binary tree T size(T) ≤ $2^{\textbf{height(T)+1}}$ -1

Languages: Sets of Strings

- Sets of strings that satisfy special properties are called *languages*. Examples:
  - English sentences
  - Syntactically correct Java/C/C++ programs
  - All strings over alphabet Σ
  - Palindromes over Σ
  - Binary strings that don't have a 0 after a 1
  - Legal variable names. keywords in Java/C/C++
  - Binary strings with an equal # of 0's and 1's (HW6)

Regular expressions

- Regular expressions over Σ
- Basis:
  - $\varnothing$, λ are regular expressions
  - ***a*** is a regular expression for any $a \in \Sigma$
- Recursive step:
  - If **A** and **B** are regular expressions then so are:
    - (**A** $\cup$ **B**)
    - (**AB**)
    - **A***

Each regular expression is a "pattern"

- λ matches the empty string
- ***a*** matches the one character string *a*
- (**A** $\cup$ **B**) matches all strings that either **A** matches or **B** matches (or both)
- (**AB**) matches all strings that have a first part that **A** matches followed by a second part that **B** matches
- **A*** matches all strings that have any number of strings (even 0) that **A** matches, one after another

Examples

- ***0****
- ***0*1****
- (***0*** $\cup$ ***1***)*
- (***0*1****)*
- (***0*** $\cup$ ***1***)* ***0110*** (***0*** $\cup$ ***1***)*
- (***0*** $\cup$ ***1***)* (***0110*** $\cup$ ***100***)(***0*** $\cup$ ***1***)*

Regular expressions in practice

- Used to define the "tokens": e.g., legal variable names, keywords in programming languages and compilers
- Used in **`grep`**, a program that does pattern matching searches in UNIX/LINUX
- Pattern matching using regular expressions is an essential feature of hypertext scripting language PHP used for web programming
  – Also in text processing programming language Perl

Regular Expressions in PHP

- int **preg_match** (string $pattern , string $subject,...)
- $pattern syntax:

`[01]` a 0 or a 1 **^** start of string **$** end of string
`[0-9]` any single digit **\.** period **\,** comma **\-** minus
. any single character

ab	a followed by b	(**AB**)
(a|b)	a or b	(**A** $\cup$ **B**)
a**?**	zero or one of a	(**A** $\cup$ λ)
a*****	zero or more of a	**A***
a**+**	one or more of a	**AA***

- e.g. `^[\-+]?[0-9]*(\.|\,)?[0-9]+$`
  General form of decimal number e.g. 9.12 or -9,8 (Europe)

More examples

- All binary strings that have an even # of 1's

- All binary strings that *don't* contain 101

Regular expressions can't specify everything we might want

- Even some easy things like palindromes
- More complicated structures in programming languages
  - Matched parentheses
  - Properly formed arithmetic expressions
  - Etc.

Context Free Grammars

- A Context-Free Grammar (CFG) is given by a finite set of substitution rules involving
  - A finite set **V** of *variables* that can be replaced
  - Alphabet Σ of *terminal symbols* that can't be replaced
  - One variable, usually **S**, is called the *start symbol*

- The rules involving a variable **A** are written as
  $\mathbf{A} \rightarrow w_1 \mid w_2 \mid \ldots \mid w_k$ where each w_i is a string of variables and terminals – that is $w_i \in (\mathbf{V} \cup \Sigma)^*$

How Context-Free Grammars generate strings

- Begin with start symbol **S**
- If there is some variable **A** in the current string you can replace it by one of the w's in the rules for **A**
  - Write this as $x\mathbf{A}y \Rightarrow xwy$
  - Repeat until no variables left
- The set of strings the CFG generates are all strings produced in this way that have no variables

Sample Context-Free Grammars

- Example: $\mathbf{S} \rightarrow 0\mathbf{S}0 \mid 1\mathbf{S}1 \mid 0 \mid 1 \mid \lambda$

- Example: $\mathbf{S} \rightarrow 0\mathbf{S} \mid \mathbf{S}1 \mid \lambda$

Sample Context-Free Grammars

- Grammar for $\{0^n1^n : n \geq 0\}$ all strings with same # of 0's and 1's with all 0's before 1's.

- Example: $\mathbf{S} \rightarrow (\mathbf{S}) \mid \mathbf{SS} \mid \lambda$

Simple Arithmetic Expressions

E $\rightarrow$ **E**+**E** | **E**$*$**E** | (**E**) | x | y | z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Generate (2$*$x) + y

Generate x+y$*$z in two fundamentally different ways

Context-Free Grammars and recursively-defined sets of strings

- A CFG with the start symbol **S** as its only variable recursively defines the set of strings of terminals that **S** can generate
- A CFG with more than one variable is a simultaneous recursive definition of the sets of strings generated by *each* of its variables
  - Sometimes necessary to use more than one

Building in Precedence in Simple Arithmetic Expressions

- **E** – expression (start symbol)
- **T** – term **F** – factor **I** – identifier **N** - number

E $\rightarrow$ **T** | **E**+**T**

T $\rightarrow$ **F** | **F**$*$**T**

F $\rightarrow$ (**E**) | **I** | **N**

I $\rightarrow$ x | y | z

N $\rightarrow$ 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Another name for CFGs

- BNF (Backus-Naur Form) grammars
  - Originally used to define programming languages
  - Variables denoted by long names in angle brackets, e.g.
    - <identifier>, <if-then-else-statement>, <assignment-statement>, <condition>
    - ::= used instead of $\rightarrow$

BNF for C

```
statement:
  ((identifier | "case" constant-expression | "default") ":")*
  (expression? ";" |
   block |
   "if" "(" expression ")" statement |
   "if" "(" expression ")" statement "else" statement |
   "switch" "(" expression ")" statement |
   "while" "(" expression ")" statement |
   "do" statement "while" "(" expression ")" ";" |
   "for" "(" expression? ";" expression? ";" expression? ")" statement |
   "goto" identifier ";" |
   "continue" ";" |
   "break" ";" |
   "return" expression? ";"
  )

block: "{" declaration* statement* "}"

expression:
  assignment-expression%

assignment-expression: (
    unary-expression (
      "=" | "*=" | "/=" | "%=" | "+=" | "-=" | "<<=" | ">>=" | "&=" |
      "^=" | "|="
    )
  )* conditional-expression

conditional-expression:
  logical-OR-expression ( "?" expression ":" conditional-expression )?
```


Parse Trees

Back to middle school:

<sentence>::=<noun phrase><verb phrase>

<noun phrase>::=<article><adjective><noun>

<verb phrase>::=<verb><adverb>|<verb><object>

<object>::=<noun phrase>

Parse:

The yellow duck squeaked loudly

The red truck hit a parked car