CSE 311 Foundations of Computing I

Autumn 2012, Lecture 3

Propositional Logic, Proofs, Predicate Calculus

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Administrative

- Course web: http://www.cs.washington.edu/311
  - Homework, Lecture slides, Office Hours ...
- Homework:
  - Due Wednesday at the start of class

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Highlights of week 1

- Propositional calculus
- Basic logical connectives
- If pigs can whistle, then horses can fly

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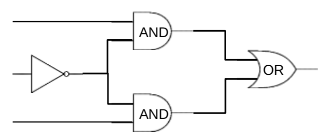
Combinational Logic Circuits



Design a 3 input circuit to compute the majority of 3. Output 1 if at least two inputs are 1, output 0 otherwise

What about a majority of 5 circuit?

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Review

Logical equivalence

- Terminology: A compound proposition is a
  - *Tautology* if it is always true
  - *Contradiction* if it is always false
  - *Contingency* if it can be either true or false

$p \vee \neg p$

$p \oplus p$

$(p \rightarrow q) \wedge p$

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

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Review

Logical Equivalence

- p and q are *logically equivalent* iff $p \leftrightarrow q$ is a tautology
  - *i.e.* p and q have the same truth table
- The notation $p \equiv q$ denotes p and q are logically equivalent
- Example: $p \equiv \neg\neg p$

p	$\neg p$	$\neg\neg p$	$p \leftrightarrow \neg\neg p$

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Review

De Morgan's Laws

- $\neg (p \wedge q) \equiv \neg p \vee \neg q$
- $\neg (p \vee q) \equiv \neg p \wedge \neg q$

- What are the negations of:
  - The Yankees and the Phillies will play in the World Series
  - It will rain today or it will snow on New Year's Day

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Review

De Morgan's Laws

Example: $\neg (p \wedge q) \equiv (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T						
T	F						
F	T						
F	F						

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Law of Implication

Example: $(p \rightarrow q) \equiv (\neg p \vee q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

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Understanding connectives

- Reflect basic rules of reasoning and logic
- Allow manipulation of logical formulas
  - Simplification
  - Testing for equivalence
- Applications
  - Query optimization
  - Search optimization and caching
  - Artificial Intelligence
  - Program verification

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Properties of logical connectives

- Identity
- Domination
- Idempotent
- Commutative
- Associative
- Distributive
- Absorption
- Negation
- DeMorgan's Laws
- Double Negation

Textbook, 1.3, Table 6

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Equivalences relating to implication

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- $p \vee q \equiv \neg p \rightarrow q$
- $p \wedge q \equiv \neg (p \rightarrow \neg q)$
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
- $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

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Logical Proofs

- To show P is equivalent to Q
  - Apply a series of logical equivalences to subexpressions to convert P to Q
- To show P is a tautology
  - Apply a series of logical equivalences to subexpressions to convert P to **T**

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Show $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

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Show $(p \rightarrow q) \rightarrow r$ and $r \rightarrow (q \rightarrow p)$ are not equivalent

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