CSE 311 Foundations of Computing I

Autumn 2012
Lecture 2
More Propositional Logic
Application: Circuits
Propositional Equivalence

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Administrative

- Course web: http://www.cs.washington.edu/311
  - Check it often: homework, lecture slides
- Office Hours: 2 × 7 = 14 hours; check the web
- Homework:
  - Paper turn-in (stapled) handed in at the **start** of class on due date (Wednesday); no online turn in.
  - Individual. OK to discuss with a couple of others but nothing recorded from discussion and write-up done much later
  - Homework 1 available (on web), due October 3

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Administrative

- Coursework and grading
  - Weekly written homework ~ 50 %
  - Midterm (November 2) ~ 15%
  - Final (December 10) ~ 35%
- A note about Extra Credit problems
  - Not required to get a 4.0
  - Recorded separately and grades calculated entirely without it
  - Fact that others do them can't lower your score
  - In total may raise grade by 0.1 (occasionally 0.2)
    - Each problem ends up worth less than required ones

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Recall…Connectives

p	$\neg p$
T	F
F	T

NOT

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

AND

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

OR

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

XOR

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$p \rightarrow q$

p	q	$p \rightarrow q$

- Implication
  - p implies q
  - whenever p is true q must be true
  - if p then q
  - q if p
  - p is sufficient for q
  - p only if q

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"If pigs can whistle then horses can fly"

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“If you behave then I’ll buy you ice cream”

What if you don’t behave?

Converse, Contrapositive, Inverse

- Implication: $p \to q$
- Converse: $q \to p$
- Contrapositive: $\neg q \to \neg p$
- Inverse: $\neg p \to \neg q$

- Are these the same?

Biconditional $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*

p	q	$p \leftrightarrow q$

English and Logic

- You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old
  - *q*: you can ride the roller coaster
  - *r*: you are under 4 feet tall
  - *s*: you are older than 16

Digital Circuits

- Computing with logic
  - **T** corresponds to 1 or “high” voltage
  - **F** corresponds to 0 or “low” voltage
- Gates
  - Take inputs and produce outputs = Functions
  - Several kinds of gates
  - Correspond to propositional connectives
    - Only symmetric ones (order of inputs irrelevant)

Gates

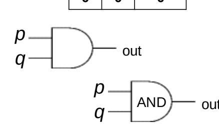
AND connective

$p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

AND gate

p	q	*out*
1	1	1
1	0	0
0	1	0
0	0	0



“block looks like D of AND”

Gates

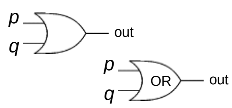
OR connective

$p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

OR gate

p	q	*out*
1	1	1
1	0	1
0	1	1
0	0	0



"arrowhead block looks like V"

Gates

NOT connective

$\neg p$

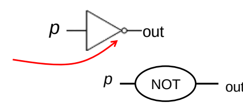
p	$\neg p$
T	F
F	T

NOT gate

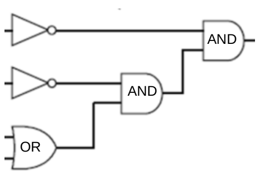
(inverter)

p	*out*
1	0
0	1

Bubble most important for this diagram

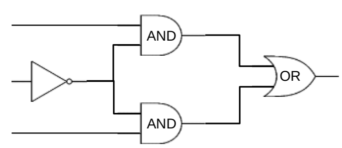


Combinational Logic Circuits



Values get sent along wires connecting gates

Combinational Logic Circuits



Wires can send one value to multiple gates

Logical equivalence

- Terminology: A compound proposition is a
  - *Tautology* if it is always true
  - *Contradiction* if it is always false
  - *Contingency* if it can be either true or false

$p \vee \neg p$

$p \oplus p$

$(p \rightarrow q) \wedge p$

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

Logical Equivalence

- p and q are *logically equivalent* iff $p \leftrightarrow q$ is a tautology
  - *i.e.* p and q have the same truth table
- The notation $p \equiv q$ denotes p and q are logically equivalent
- Example: $p \equiv \neg\neg p$

p	$\neg p$	$\neg\neg p$	$p \leftrightarrow \neg\neg p$

De Morgan's Laws

$\neg (p \wedge q) \equiv \neg p \vee \neg q$
$\neg (p \vee q) \equiv \neg p \wedge \neg q$

What are the negations of:
- The Yankees and the Phillies will play in the World Series
- It will rain today or it will snow on New Year's Day

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De Morgan's Laws

Example: $\neg (p \wedge q) \equiv (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T						
T	F						
F	T						
F	F						

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Law of Implication

Example: $(p \rightarrow q) \equiv (\neg p \vee q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

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Computing equivalence

- Describe an algorithm for computing if two logical expressions/circuits are equivalent
- What is the run time of the algorithm?

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Understanding connectives

- Reflect basic rules of reasoning and logic
- Allow manipulation of logical formulas
  - Simplification
  - Testing for equivalence
- Applications
  - Query optimization
  - Search optimization and caching
  - Artificial Intelligence
  - Program verification

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Properties of logical connectives

- Identity
- Domination
- Idempotent
- Commutative
- Associative
- Distributive
- Absorption
- Negation

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Equivalences relating to implication

- $p \to q \equiv \neg p \lor q$
- $p \to q \equiv \neg q \to \neg p$
- $p \lor q \equiv \neg p \to q$
- $p \land q \equiv \neg (p \to \neg q)$
- $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
- $\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

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Logical Proofs

- To show P is equivalent to Q
  - Apply a series of logical equivalences to subexpressions to convert P to Q
- To show P is a tautology
  - Apply a series of logical equivalences to subexpressions to convert P to **T**

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Show $(p \land q) \to (p \lor q)$ is a tautology

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