CSE311 Quiz Section: September 27, 2012

1 Introductions

Let's all introduce ourselves!

2 Knights and Knaves

You are on an island of knights and knaves. Knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what the two people are:

1. A says "At least one of us is a knave" and B says nothing.

   A is a knight and B is a knave.

2. A says "The two of us are both knights" and B says "A is a knave"

   A is a knave and B is a knight.

3. A says "I am a knave or B is a knight" and B says nothing.

   Both are knights.

4. Both A and B say "I am a knight"

   Either one could either thing.

3 Implication operator: $p \to q$

Truth Table:

p	q	$p \to q$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

You can write $p \to q$ as $\neg p \vee q$. Can you see why? Compare the truth tables.

4 Implications

Express each of these statements in the form "if p, then q" in English.

1. It snows whenever the wind blows from the northeast.

   If it blows from the Northeast, then it snows.

2. The apple trees will bloom if it stays warm for a week.

   If it stays warm for a week, then the apple trees will bloom.

3. That the Pistons win the championship implies that they beat the Lakers.

   If the Pistons are the champions, then they must have beat the Lakers.

4. It is necessary to walk 8 miles to get to the top of Long's Peak.

   If you are at Long's Peak, then you must have walked 8 miles.

5. To get tenure as a professor, it is sufficient to be world-famous.

   If you are world-famous, then you can get tenure as a professor.

6. Your guarantee is good only if you bought your CD player less than 90 days ago.

   If your guarantee is good, then you must have bought your CD player less than 90 days ago.

   For the last one:
   Some students thought the implication goes the opposite way. Any easy way to see why that is not the case is the following: Suppose that for your guarantee to be good, you must fulfill a set of conditions. One of them is the 90 days condition. For the sake of the argument, suppose that the other one is that you must be at least 25 years old. Then notice that the fact that you bought your CD player less than 90 days ago does not suffice for your guarantee to be good. On the other hand, if your guarantee is good the it must mean that you bought the CD player at most 90 days ago.

5 De Morgan's Law

We know that we can rewrite $p \rightarrow q$ as $\neg p \vee q$. What is $\neg(p \rightarrow q)$ equivalent to? Use De Morgan's law. Intuitively, explain what is going on.

We know that the only assigment of values to p, q for which the implication is false, is when p is True and q is False. We have that:

$$\neg(\neg p \vee q) \equiv \neg(\neg p) \wedge \neg q \equiv p \wedge \neg q$$

This is a compound proposition created by conjoining $p, \neg q$, by the logical AND. Therefore it is bound to be True only for a specific assignment of values and that is $p =$True$, q =$False, which is precisely when the implication is False.