CSE 311 Quiz Section: October 18, 2012 (Solutions)

2 More on sets.

Prove that $A \subseteq B \leftrightarrow \bar{B} \subseteq \bar{A}$.

Proof. (For a biconditional statement $P \leftrightarrow Q$, we must show both that $P \rightarrow Q$ and $Q \rightarrow P$ in order to complete the proof.)

($\rightarrow$) Let $A \subseteq B$, and suppose $x \in \bar{B}$.
Then $x \notin B$ by definition of set complements.
Since $A \subseteq B$, then $\forall y(y \in A \rightarrow y \in B)$, or $\forall y(y \notin B \rightarrow y \notin A)$ [contrapositive], so it follows that $x \notin A$.
Therefore $x \in \bar{A}$ by def. of set complements.
Since we have shown that $x \in \bar{B} \rightarrow x \in \bar{A}$, then $\bar{B} \subseteq \bar{A}$ by definition of subset.

($\leftarrow$) Let $\bar{B} \subseteq \bar{A}$, and suppose $x \in A$. By a symmetrical argument, $x \in B$. Thus $A \subseteq B$.
$\square$

3 Memories of functions.

For all functions and mappings below, state whether they are injective (one-to-one), surjective (onto), or bijective (both) over the following sets:

$A = \{x | x \in \mathbb{R}, x \geq 1\}$
$B = \{x | x \in \mathbb{R}, 0 \leq x \leq 1\}$
$C = \{x | x \in \mathbb{R}, -1 \leq x \leq 1\}$

1. $f\colon A \rightarrow B$, $f(x) = \frac{1}{x}$
   Answer: Injective, but not surjective ($0 \in B$, but $\frac{1}{x} \neq 0$ $\forall x \in A$.)

2. $f\colon B \rightarrow C$, $f(x) = x^2$
   Answer: Injective, but not surjective ($-1 \in C$, but $x^2 \neq -1$ $\forall x \in B$.)
3. $f\colon B \rightarrow B$, $f(x) = x^2$
   Answer: Both one-to-one and onto, so bijective. (No negatives to worry about in this case, so we don't have the same problem as 2 for surjective or the same problem as 4 for injective.)
4. $f\colon C \rightarrow B$, $f(x) = x^2$
   Answer: Surjective, but not injective. ($f(-1) = f(1) = 1$, but $-1 \neq 1$)

4 Modular Arithmetic.

Find $a \in \mathbb{Z}$ such that:

1. $a \equiv 43 \pmod{23}$, $-22 \leq a \leq 0$
   Answer: $a = -3$ (we can check by seeing that $23|(43-(-3))$)
2. $a \equiv 17 \pmod{29}$, $-14 \leq a \leq 14$
   Answer: $a = -12$ (we can check by seeing that $29|(17-(-12))$)
3. $a \equiv -11 \pmod{21}$, $90 \leq a \leq 110$
   Answer: $a = 94$ (we can check by seeing that $21|(94-(-11))$)