CSE311 Quiz Section: October 11, 2012 (Solutions)

2 Logical equivalence with quantifiers

Solutions in book: 7th ed- Section 1.4 Problems 43, 45; 6th ed- Section 1.3 Problems 43, 45

3 Use inference rules with quantified premises and conclusions

Solutions in book: 7th ed- Section 1.6 Problems 27, 29; 6th ed- Section 1.5 Problems 27, 29

4 Extra: Prove that the square of a natural number n is greater than or equal to the sum of all the numbers between 1 and n (1, n included).

Proof. Let n be a natural number. Then the sum of all integers between 1 and n (inclusive) can be written:

$$1 + 2 + 3 + ... + n = \sum_{k=1}^{n} k.$$

Similarly, we can write $n^2 = \sum_{k=1}^{n} n$ (i.e. n summed with itself n times).
Since $n \geq k$ for each $1 \leq k \leq n$, then it follows that:

$$\sum_{k=1}^{n} k \leq \sum_{k=1}^{n} n = n^2.$$

□

***Note:** There are other ways to prove this. In particular, you could use theorem that states that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and compare this with n^2.