CSE 311 Quiz Section: November 1, 2012 (Solutions)

1 Using Strong Induction

Which amounts of money can be formed using just two-dollar bills and five-dollar bills? Prove your answer using strong induction.

Answer: *Solution in book: 7th edition- Sect 5.2 Problem 7; 6th edition- Section 4.2 Problem 7)*

2 Recursive functions

Find $f(1), f(2), f(3)$, and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 0, 1, 2, ...$

a) $f(n+1) = f(n) + 2$
Answer: $f(1) = 3, f(2) = 5, f(3) = 7, f(4) = 9$

b) $f(n+1) = 3f(n)$
Answer: $f(1) = 3, f(2) = 9, f(3) = 27, f(4) = 81$

c) $f(n+1) = 2^{f(n)}$
Answer: $f(1) = 2, f(2) = 4, f(3) = 16, f(4) = 2^{16}$

d) $f(n+1) = f(n)^2 + f(n) + 1$
Answer: $f(1) = 3, f(2) = 13, f(3) = 183, f(4) = 33,673$

3 Recursive proof

Prove that $f_0f_1 + f_1f_2 + ... + f_{2n-1}f_{2n} = f_{2n}^2$ where n is a positive integer and f_n is the nth Fibonacci number.

Proof. (By induction)

Base Case: Let $n = 1$.
Then $f_0f_1 + f_1f_2 = (0)(1) + (1)(1) = 1$ and
$f_2^2 = (1)(1) = 1$, therefore $f_0f_1 + f_1f_2 = f_2^2$.

Inductive Step: Assume $f_0f_1 + f_1f_2 + ... + f_{2k-1}f_{2k} = f_{2k}^2$.
Show that $f_0f_1 + ... + f_{2(k+1)-1}f_{2(k+1)} = f_{2(k+1)}^2$.

So, starting with the left hand side of our equation (and simplifying the subscripts), we have:

$$
\begin{aligned}
f_0f_1 + f_1f_2 + ... + f_{2k+1}f_{2k+2} \\
&= (f_0f_1 + f_1f_2 + ... + f_{2k-1}f_{2k}) + f_{2k}f_{2k+1} + f_{2k+1}f_{2k+2} \\
&= (f_{2k}^2) + f_{2k}f_{2k+1} + f_{2k+1}f_{2k+2} \quad \textit{(by inductive hypothesis)} \\
&= f_{2k}f_{2k} + f_{2k}f_{2k+1} + f_{2k+1}f_{2k+2} \quad \textit{(expanding squared term)} \\
&= f_{2k}(f_{2k} + f_{2k+1}) + f_{2k+1}f_{2k+2} \quad \textit{(by distributive property)} \\
&= f_{2k}f_{2k+2} + f_{2k+1}f_{2k+2} \quad \textit{(by def. of Fib. numbers)} \\
&= f_{2k+2}(f_{2k} + f_{2k+1}) \quad \textit{(by distributive property)} \\
&= f_{2k+2}f_{2k+2} \quad \textit{(by def. of Fib numbers)} \\
&= f_{2(k+1)}^2
\end{aligned}
$$

This is what we were trying to show, thus we have satisfied our inductive step and proved the statement for all integers $n \geq 1$. □